Szalinski: A Tool for Synthesizing Structured CAD Models with Equality Saturation and Inverse Transformations

PLDI 2020

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PLSE
PAUL G. ALLEN SCHOOL OF COMPUTER SCIENCE & ENGINEERING
MAX PLANCK INSTITUTE FOR SOFTWARE SYSTEMS

Designing Physical Objects

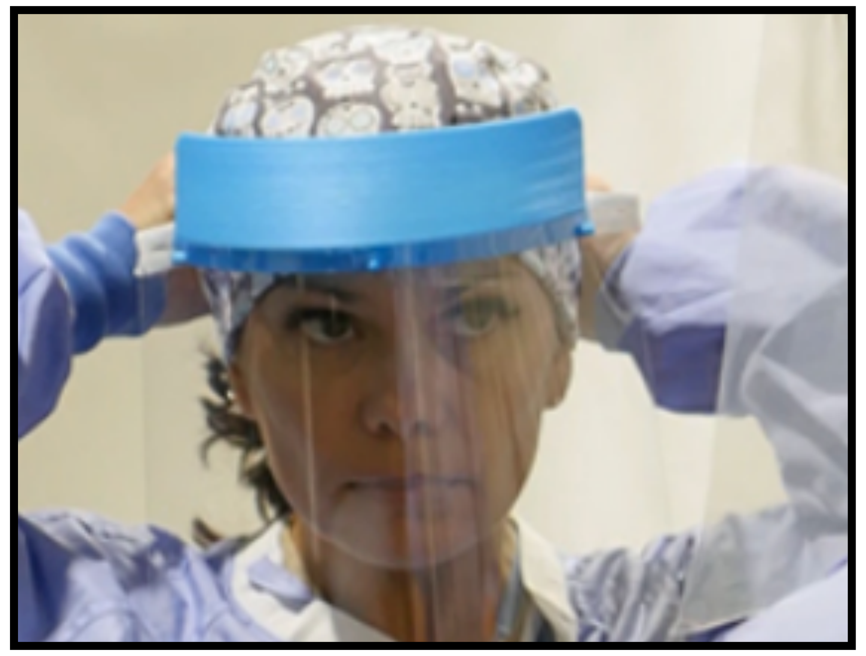
is

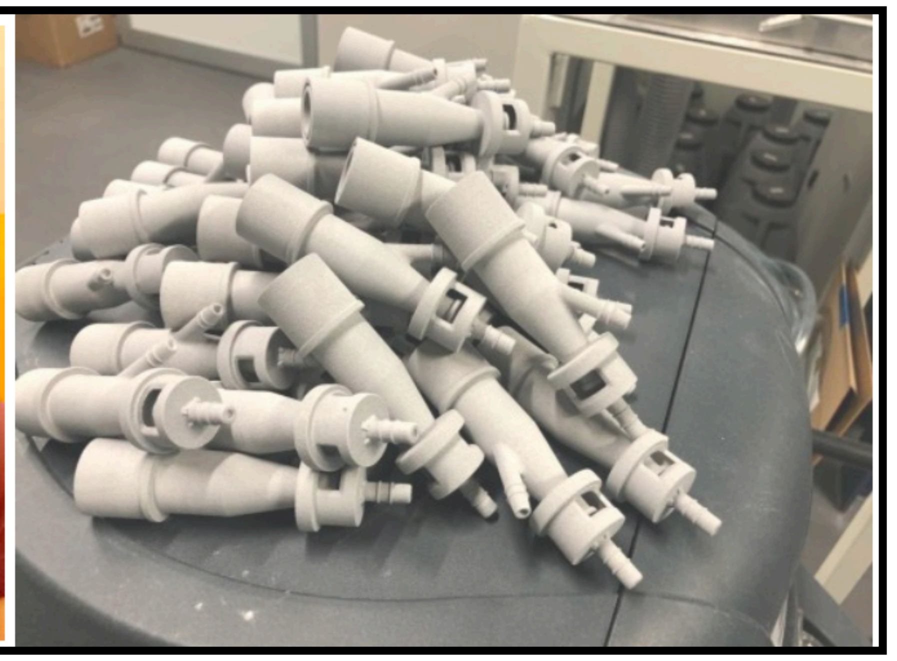
Programming!

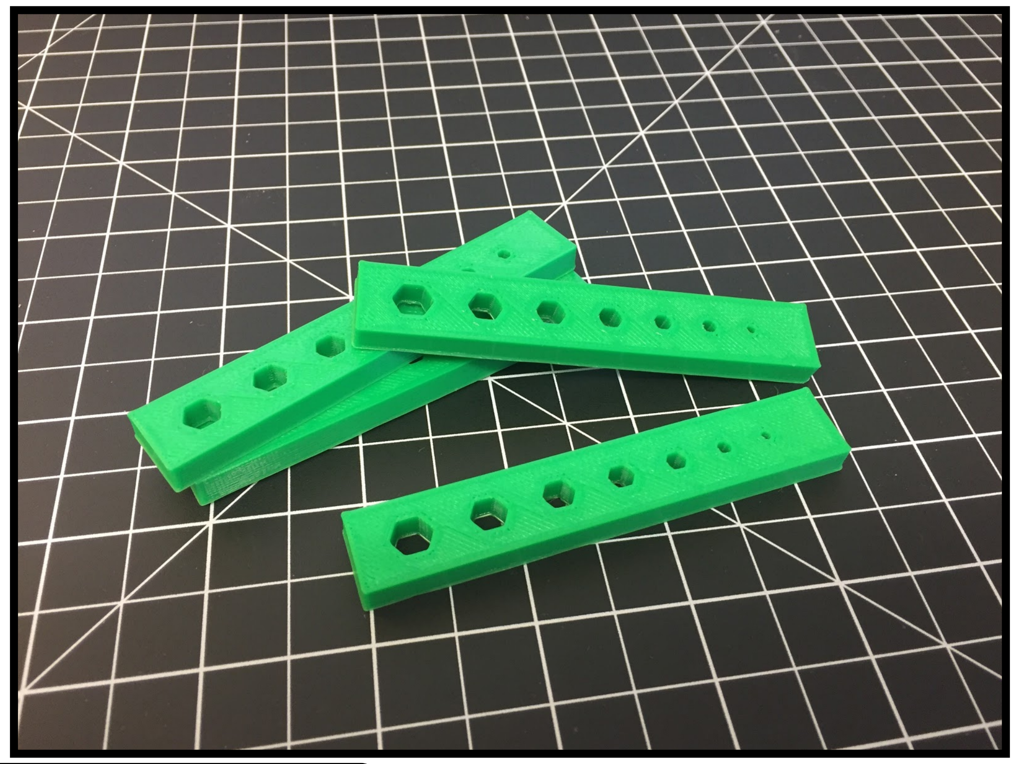
CAD and 3D Printing everywhere!

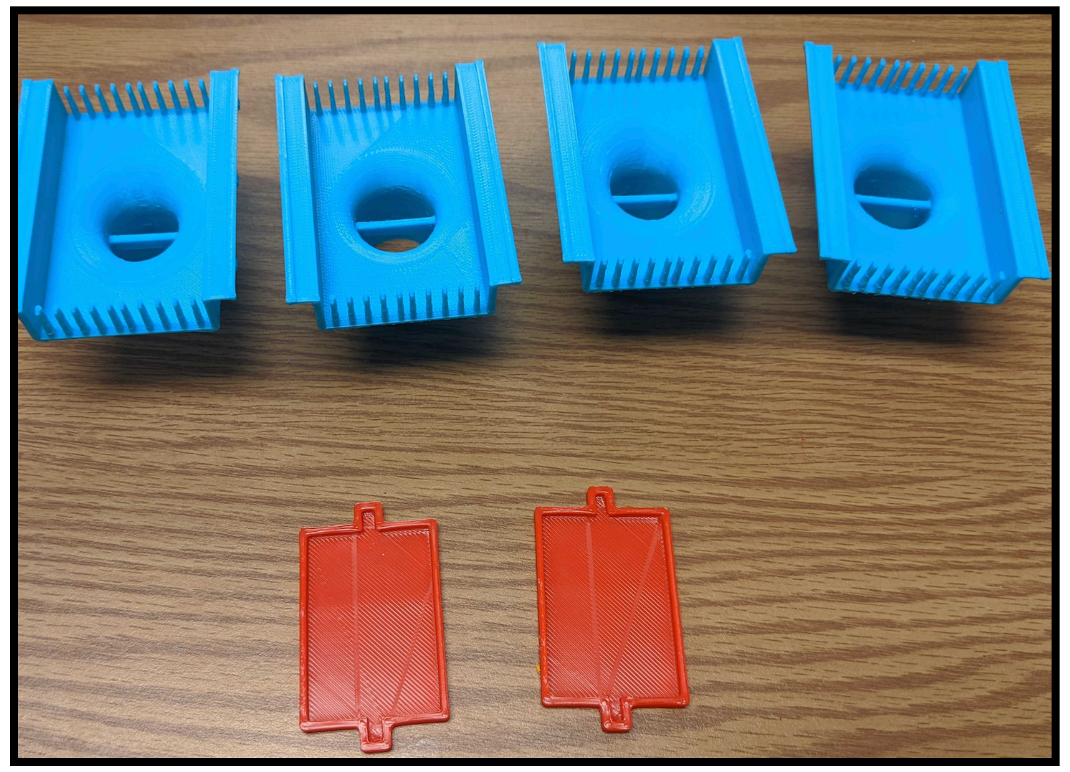
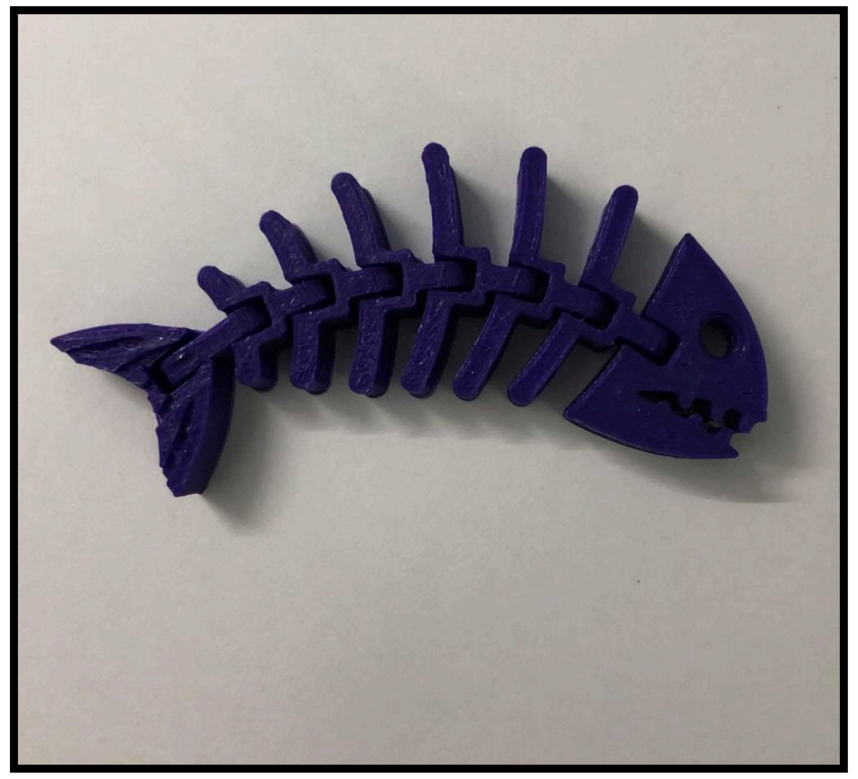
Make your own models







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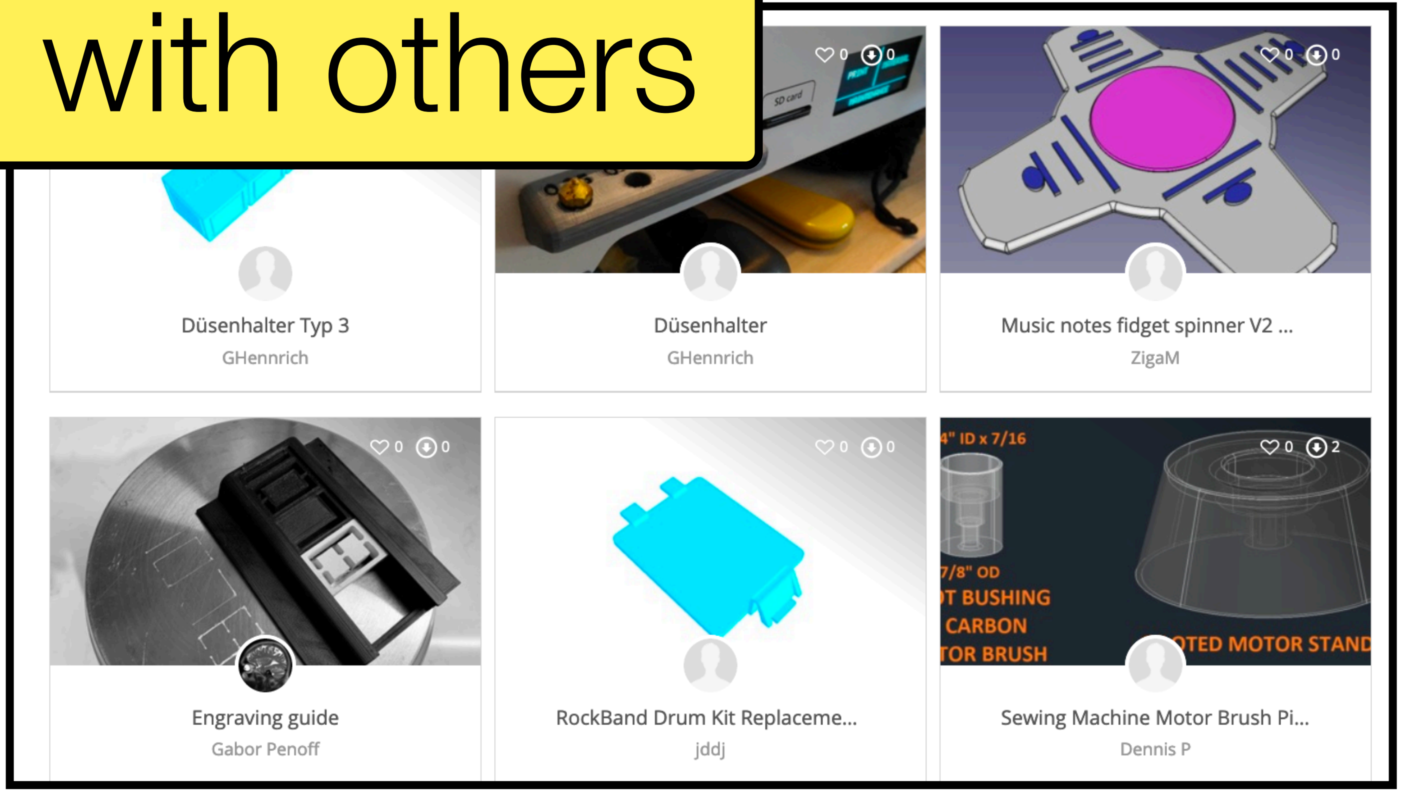
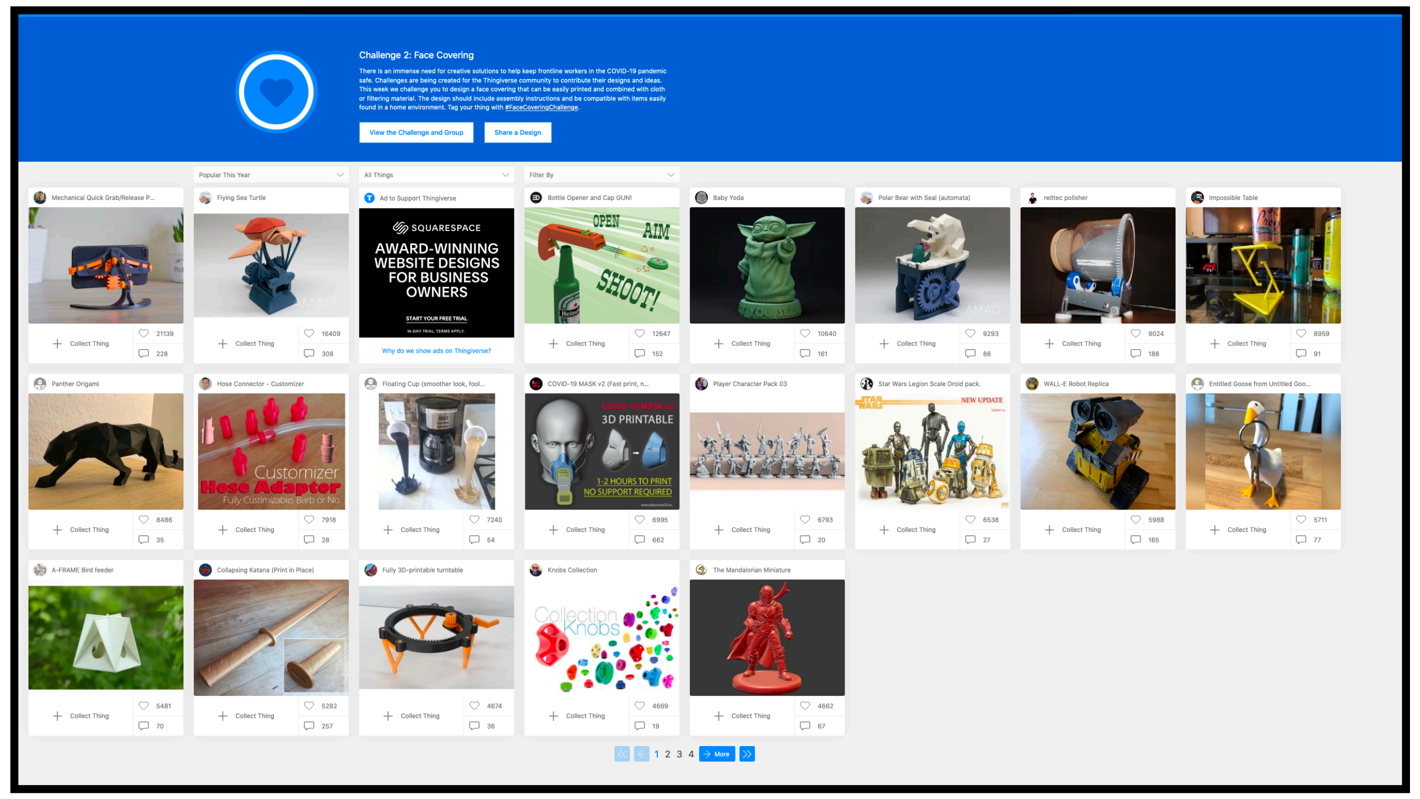
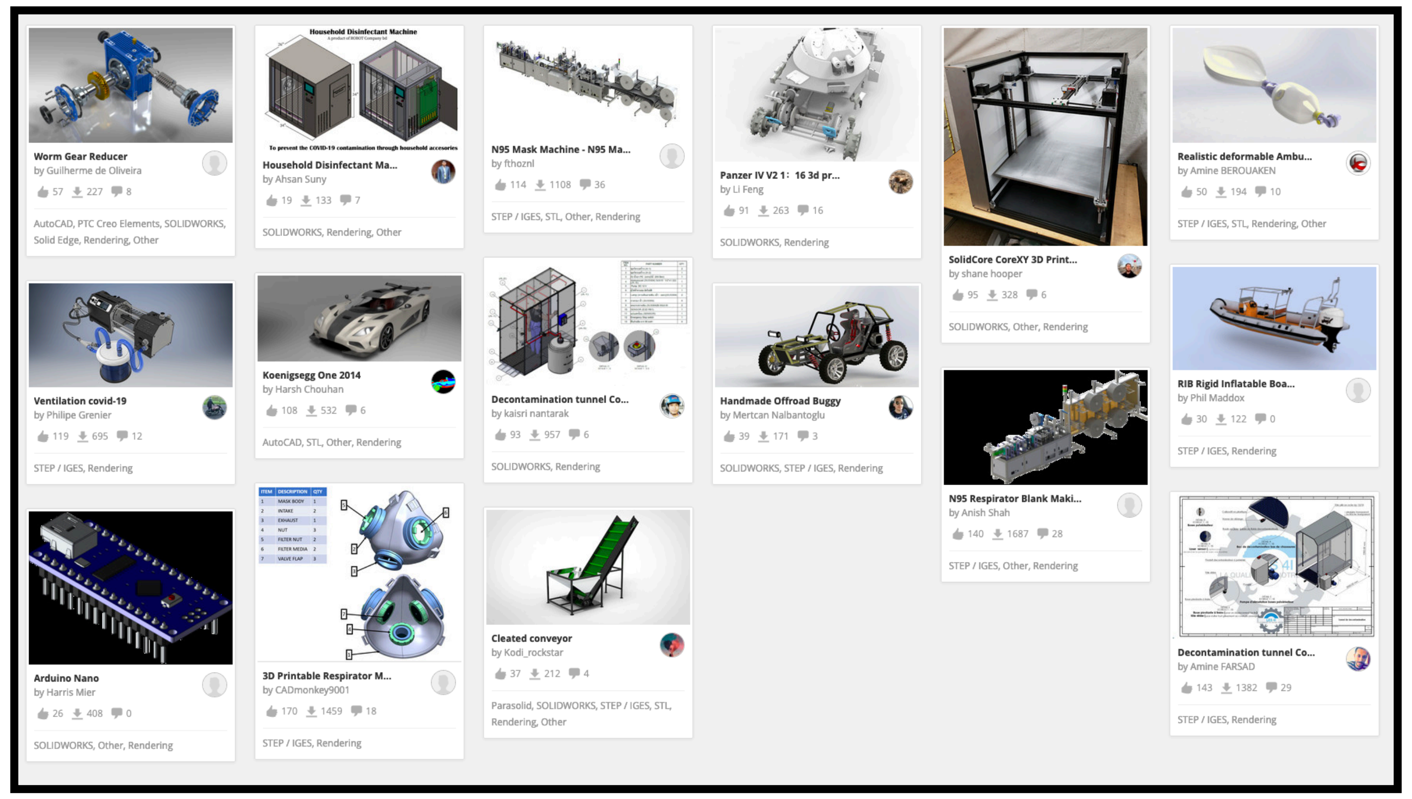
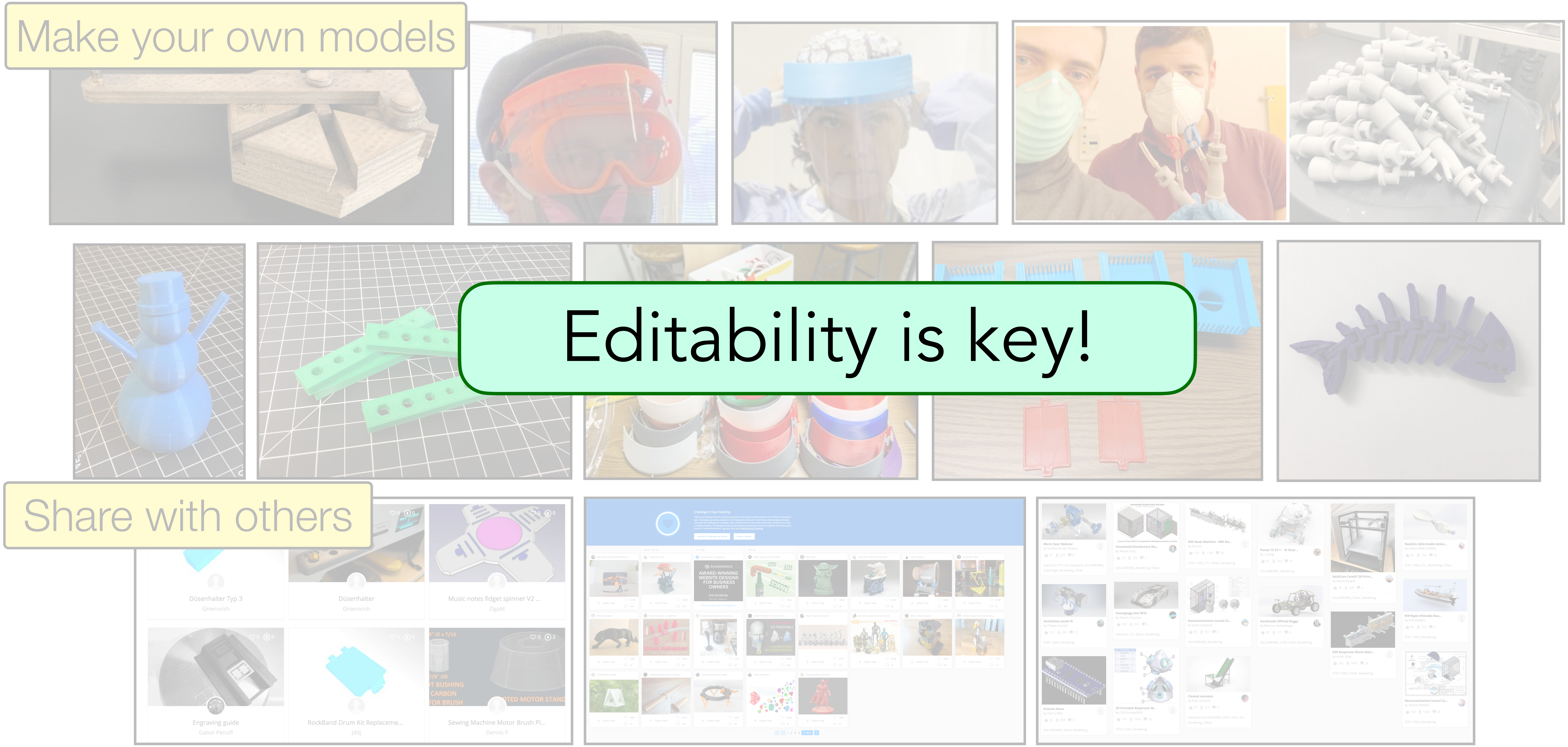


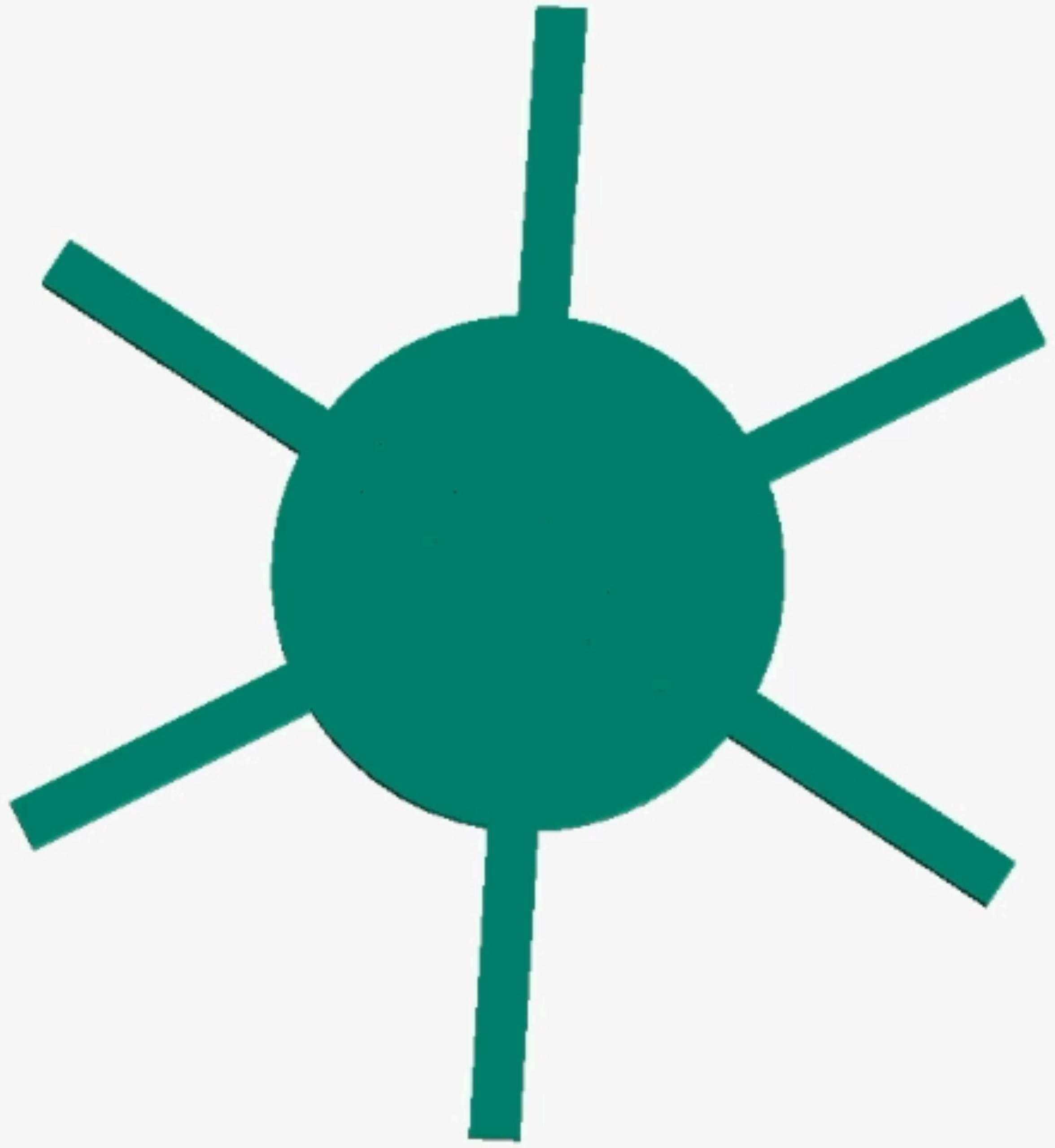
CAD and 3D Printing everywhere!

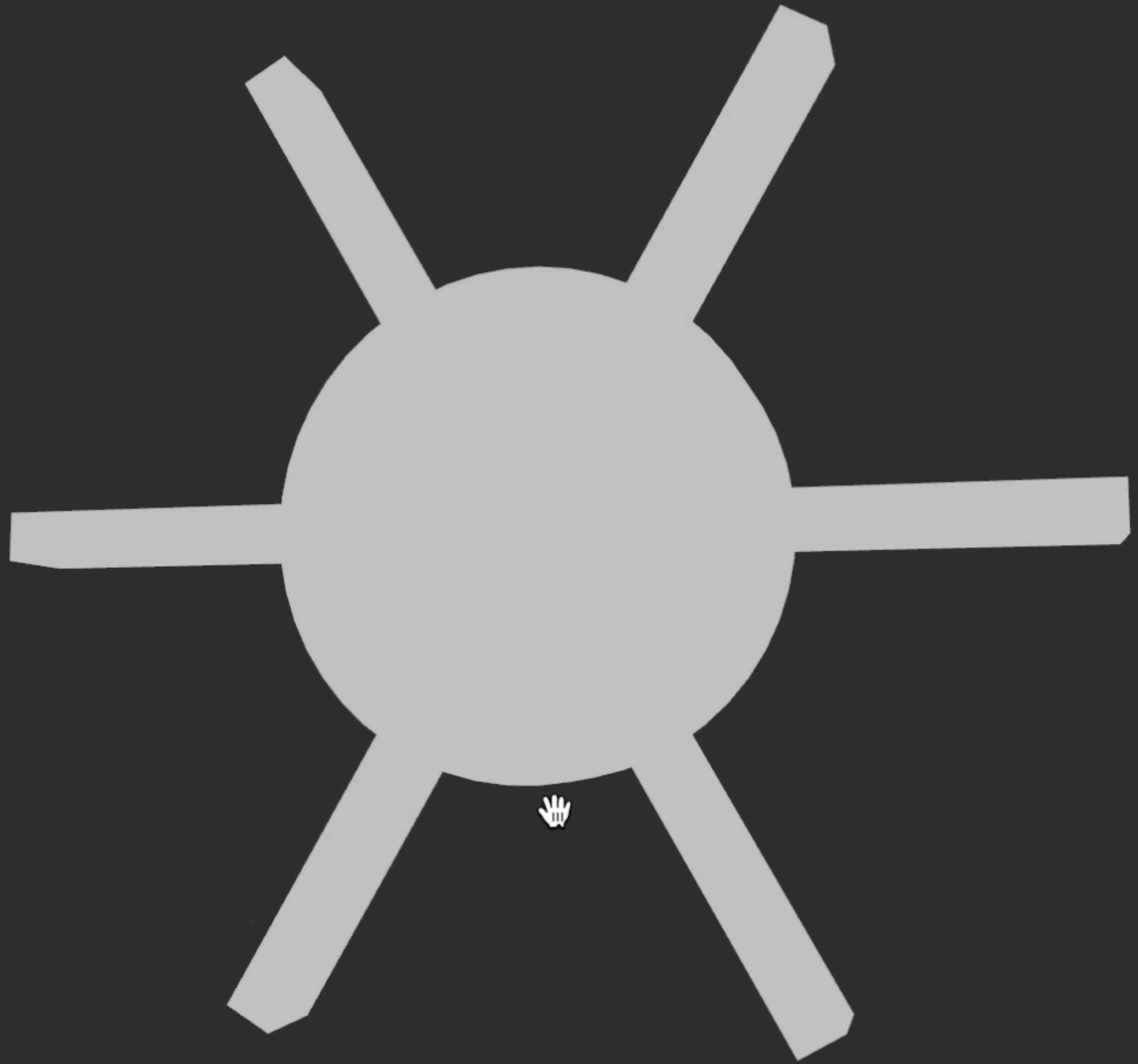


```
n = 6;

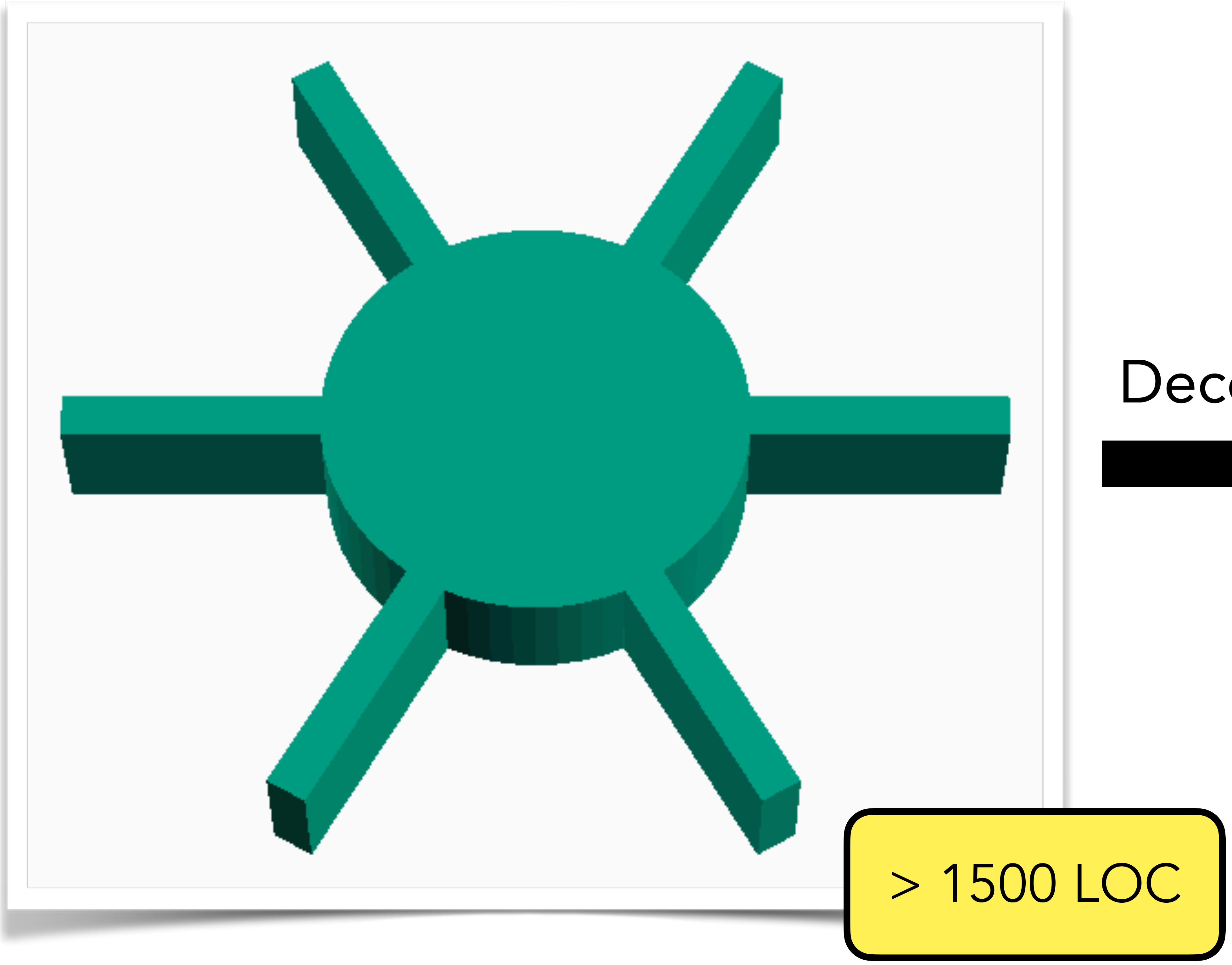
cylinder(h= 2, r=5, $fn=50);

for (i = [0:n-1]) {
    rotate([0, 0, i * 360 / n])
    translate([1, -0.5, 0])
    cube([10, 1, 2]);
}
```





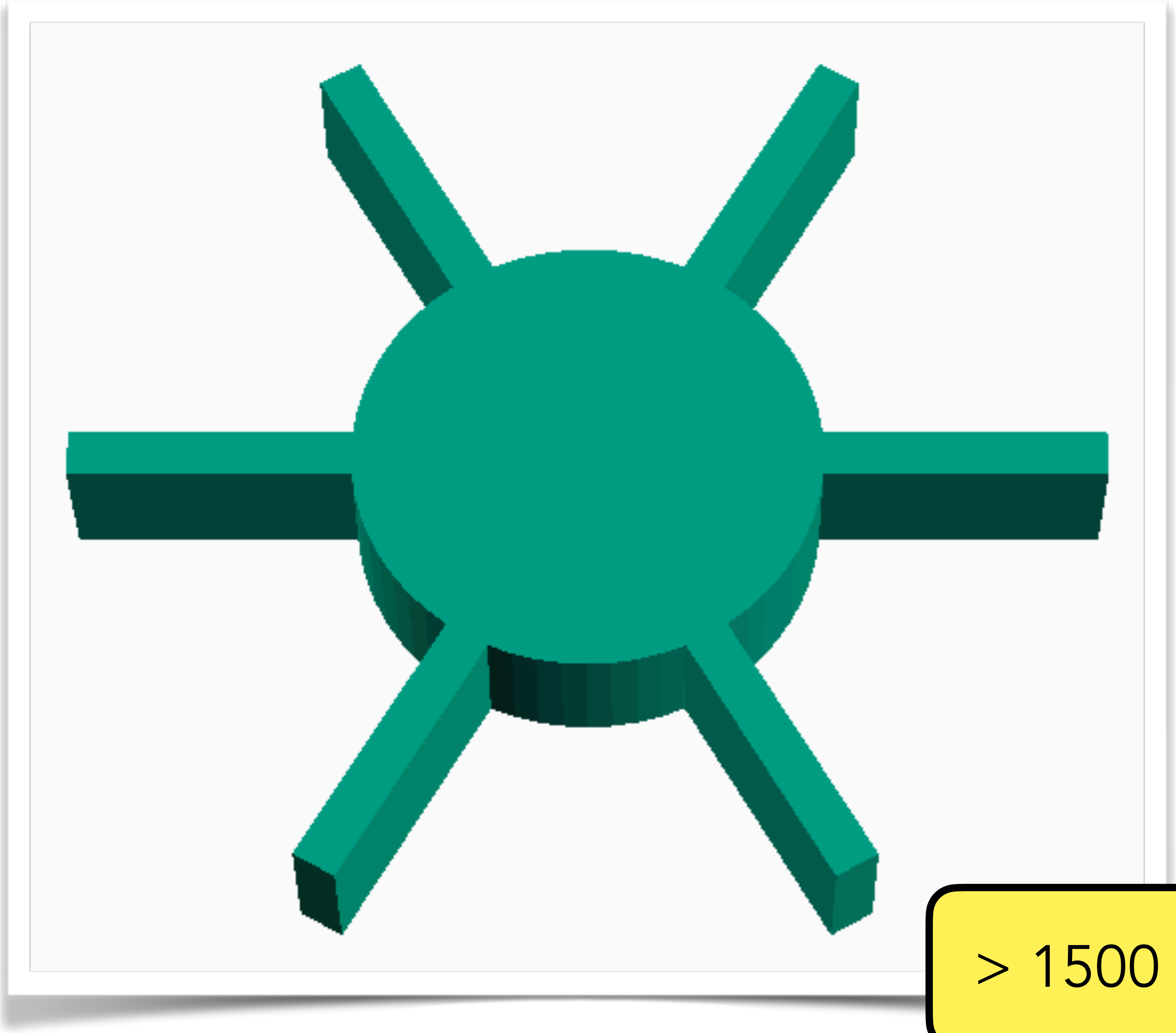
Mesh Decompilers Recover Flat Programs




```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Union
    (Rotate [0,0,120]
      (Translate [1,–0.5,0] (Cuboid [10,1,1])))
    (Scale [10,1,1]
      (Translate [0.1,–0.5,1] (Cuboid [1,1,1])))
    (Rotate [0, 0, 300]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
    (Translate [–1,0.5,0]
      (Scale [–1,–1,1] Cuboid [10,1,1]))
    (Rotate [0, 0, 240]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
    (Rotate [0, 0, 60]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))))
```

* Reincarnate [ICFP 2018], InverseCSG [SIGGRAPH Asia 2018], Shape2Prog [ICLR 2019], CSGNet [CVPR 2018], …

Mesh Decompilers Recover Flat Programs



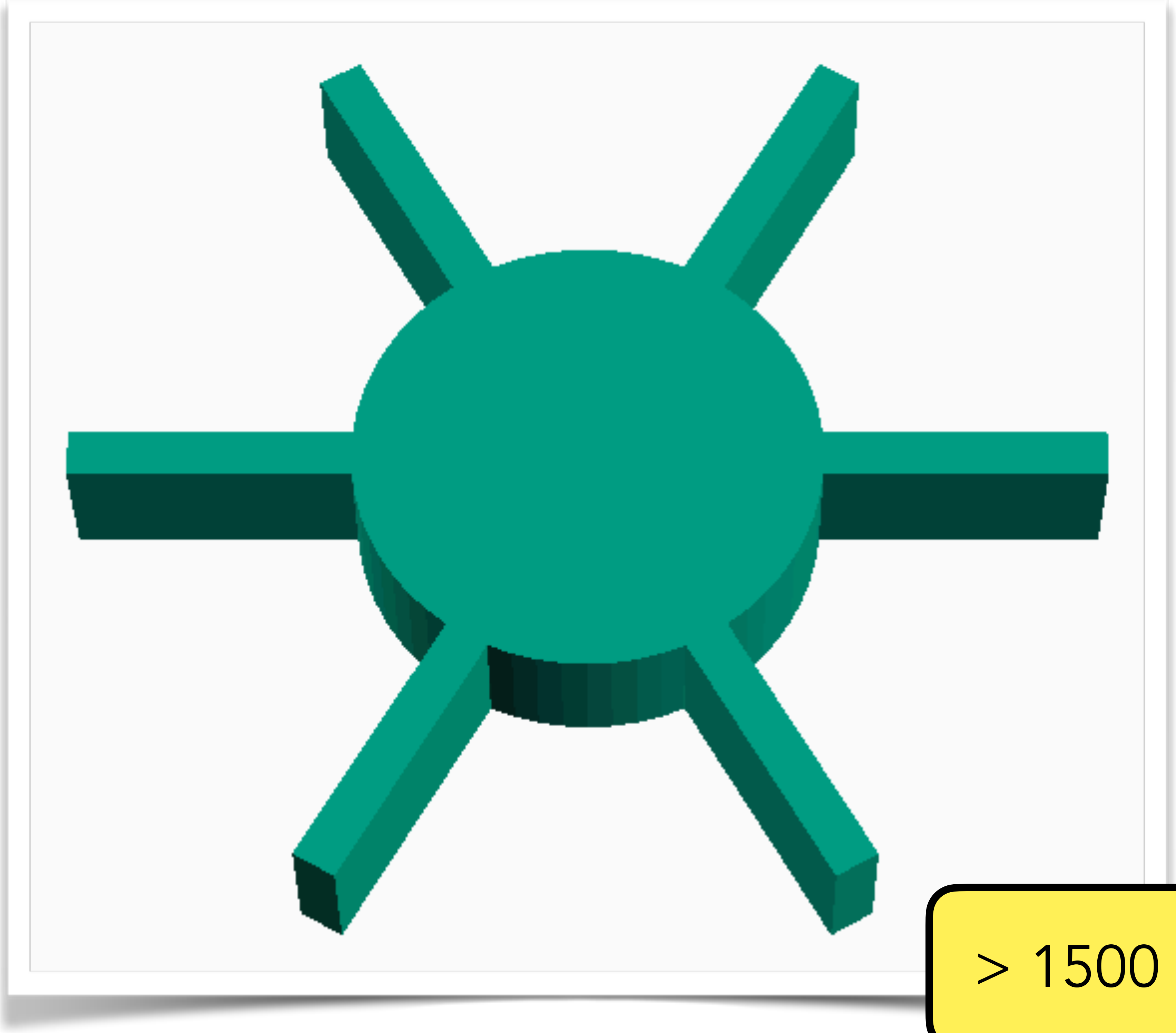
Mesh Decompilers *

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Union
    (Rotate [0,0,120]
      (Translate [1,–0.5,0] (Cuboid [10,1,1])))
    (Scale [10,1,1]
      (Translate [0.1,–0.5,1] (Cuboid [1,1,1])))
    (Rotate [0, 0, 300]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
    (Translate [–1,0.5,0]
      (Scale [–1,–1,1] Cuboid [10,1,1]))
    (Rotate [0, 0, 240]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
    (Rotate [0, 0, 60]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))))
```

Primitives

* Reincarnate [ICFP 2018], InverseCSG [SIGGRAPH Asia 2018], Shape2Prog [ICLR 2019], CSGNet [CVPR 2018], …

Mesh Decompilers Recover Flat Programs



Mesh Decompilers *

Affine operators

Primitives

> 1500 LOC

```
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 (Scale [5,5,1] (Cylinder [1,1]))
 (Union
  (Rotate [0,0,120]
   (Translate [1,−0.5,0] (Cuboid [10,1,1])))
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   (Translate [0.1,−0.5,1] (Cuboid [1,1,1])))
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   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
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  (Rotate [0, 0, 240]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))
```

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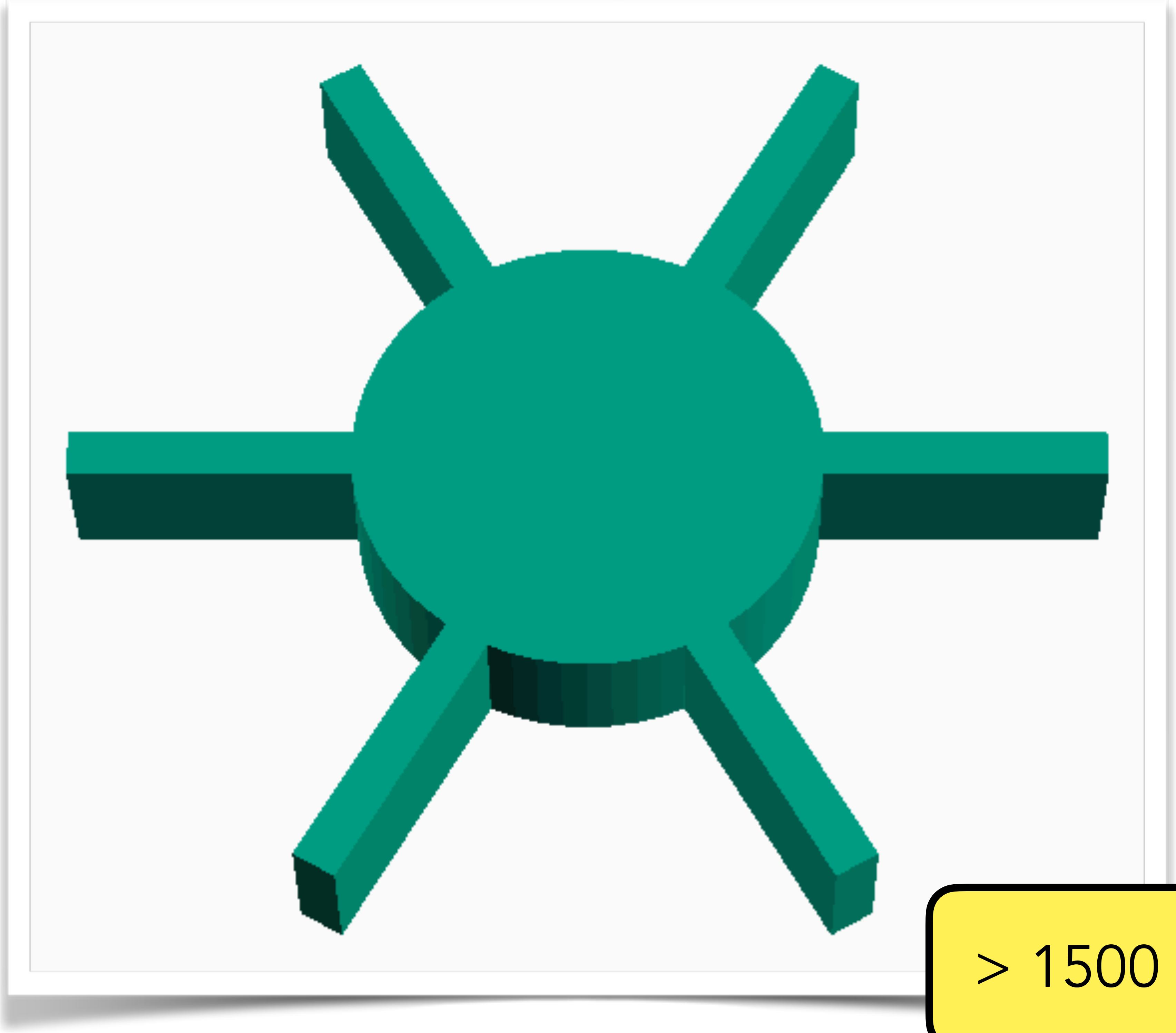
Mesh Decompilers Recover Flat Programs



> 1500 LOC

Mesh Decompilers *

Affine operators

Binary operators

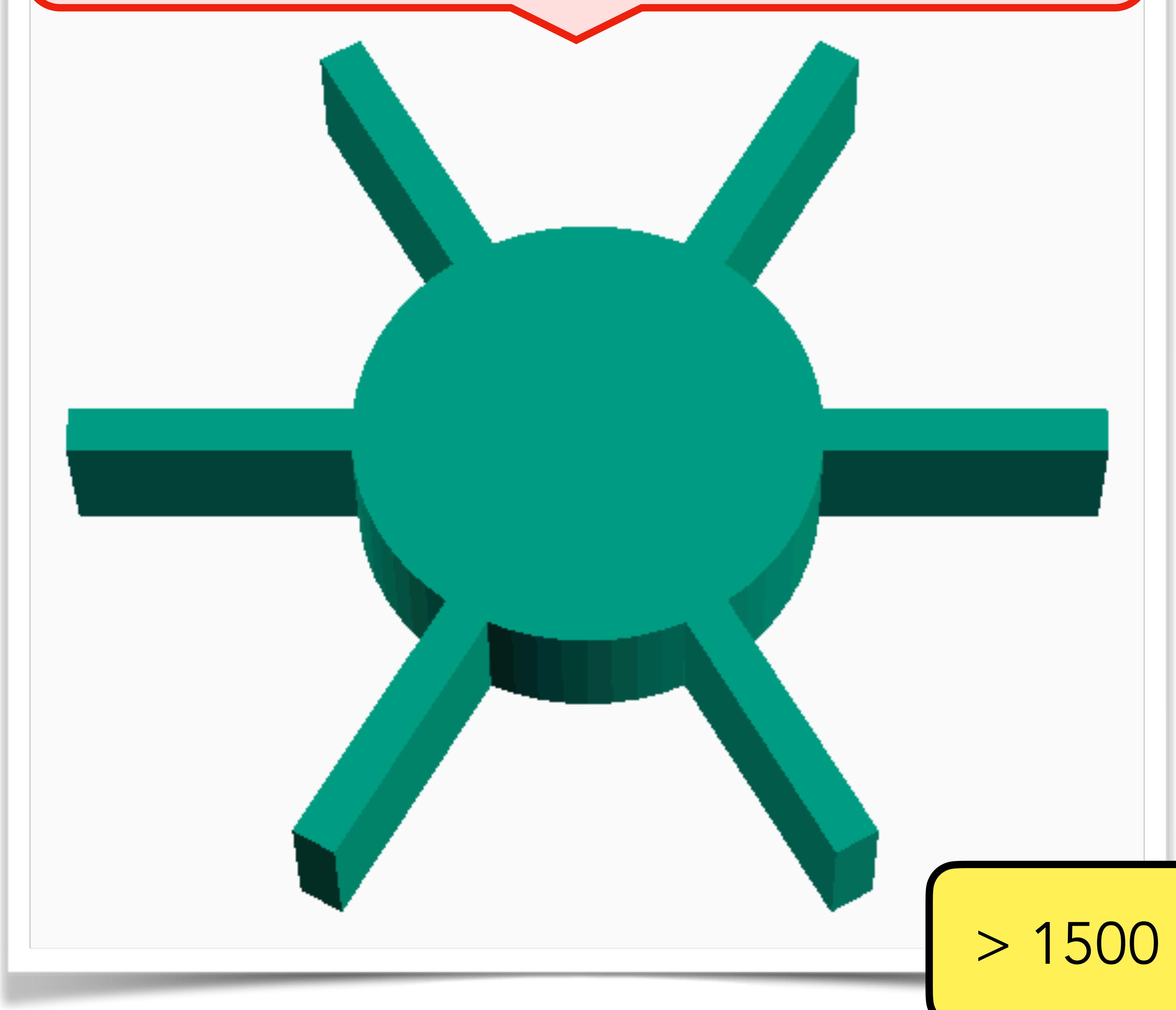
Primitives

```
(Union
 (Scale [5,5,1] (Cylinder [1,1]))
 (Union
  (Rotate [0,0,120]
   (Translate [1,–0.5,0] (Cuboid [10,1,1])))
  (Scale [10,1,1]
   (Translate [0.1,–0.5,1] (Cuboid [1,1,1])))
  (Rotate [0, 0, 300]
   (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Translate [–1,0.5,0]
   (Scale [–1,–1,1] Cuboid [10,1,1]))
  (Rotate [0, 0, 240]
   (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
   (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))))
```

* Reincarnate [ICFP 2018], InverseCSG [SIGGRAPH Asia 2018], Shape2Prog [ICLR 2019], CSGNet [CVPR 2018], …

Mesh Decompilers Recover Flat Programs

Repetition of spokes is not captured by flat program



> 1500 LOC

Mesh Decompilers *

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Union
    (Rotate [0,0,120]
      (Translate [1,–0.5,0] (Cuboid [10,1,1])))
    (Scale [10,1,1]
      (Translate [0.1,–0.5,1] (Cuboid [1,1,1])))
    (Rotate [0, 0, 300]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
    (Translate [–1,0.5,0]
      (Scale [–1,–1,1] Cuboid [10,1,1]))
    (Rotate [0, 0, 240]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
    (Rotate [0, 0, 60]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))))
```

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Szalinski: flat CAD → parametrized CAD

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Union
    (Rotate [0,0,120]
      (Translate [1,−0.5,0] (Cuboid [10,1,1])))
    (Scale [10,1,1]
      (Translate [0.1,−0.5,1] (Cuboid [1,1,1])))
    (Rotate [0, 0, 300]
      (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
    (Translate [−1,0.5,0]
      (Scale [−1,−1,1] Cuboid [10,1,1]))
    (Rotate [0, 0, 240]
      (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
    (Rotate [0, 0, 60]
      (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))
```

Szalinski →

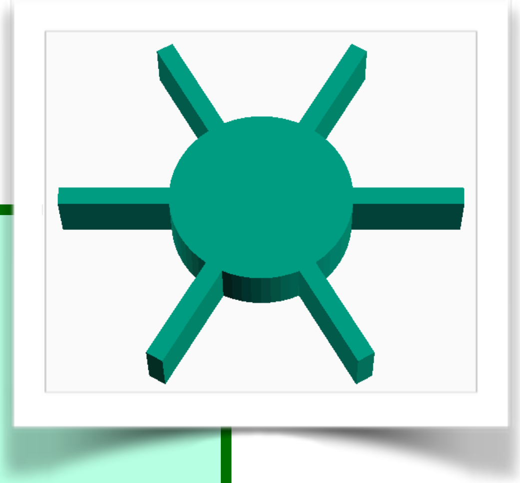
This talk

Szalinski: flat CAD → parametrized CAD

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Union
    (Rotate [0,0,120]
      (Translate [1,–0.5,0] (Cuboid [10,1,1])))
    (Scale [10,1,1]
      (Translate [0.1,–0.5,1] (Cuboid [1,1,1])))
    (Rotate [0, 0, 300]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
    (Translate [−1,0.5,0]
      (Scale [−1,−1,1] Cuboid [10,1,1]))
    (Rotate [0, 0, 240]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
    (Rotate [0, 0, 60]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1]))))
```

Szalinski →

This talk



```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,–0.5,0]
     (Cuboid [10, 1, 1]))))))
```

Szalinski: flat CAD → parametrized CAD

```
(Union
 (Scale [5,5,1] (Cylinder [1,1]))
 (Union
  (Rotate [0,0,120]
   (Translate [1,–0.5,0] (Cuboid [10,1,1])))
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   (Translate [0.1,–0.5,1] (Cuboid [1,1,1])))
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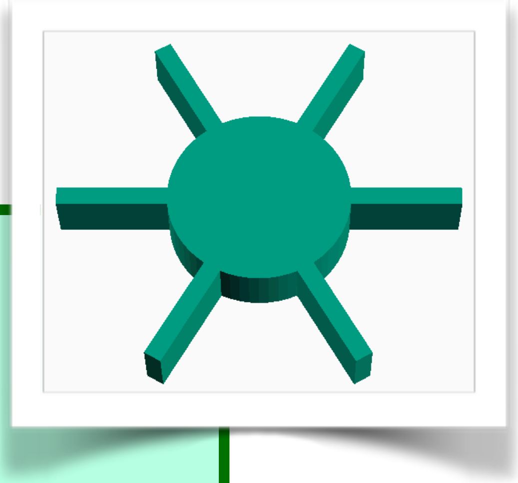
   (Scale [−1,−1,1] Cuboid [10,1,1]))
  (Rotate [0, 0, 240]
   (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
   (Translate [1, –0.5, 0] (Cuboid [10, 1, 1]))))
```

Szalinski

This talk



```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,–0.5,0]
     (Cuboid [10, 1, 1]))))))
```

A language, called *Caddy* that supports CAD features & functional programming features like Fold, Tabulate, Map

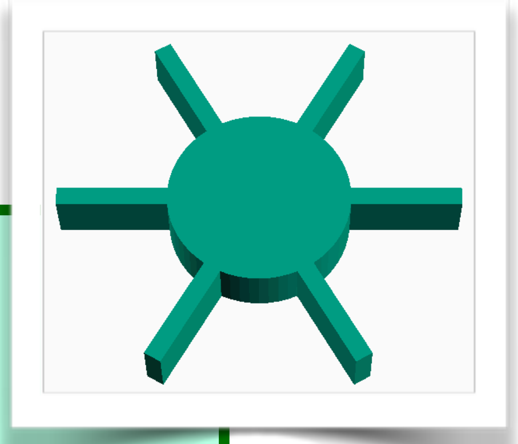
Fold and Tabulate represent loops

Szalinski: flat CAD → parametrized CAD

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Union
    (Rotate [0,0,120]
      (Translate [1,–0.5,0] (Cuboid [10,1,1])))
    (Scale [10,1,1]
      (Translate [0.1,–0.5,1] (Cuboid [1,1,1])))
    (Rotate [0, 0, 300]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
    (Translate [–1,0.5,0]
      (Scale [–1,–1,1] Cuboid [10,1,1]))
    (Rotate [0, 0, 240]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
    (Rotate [0, 0, 60]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))))
```

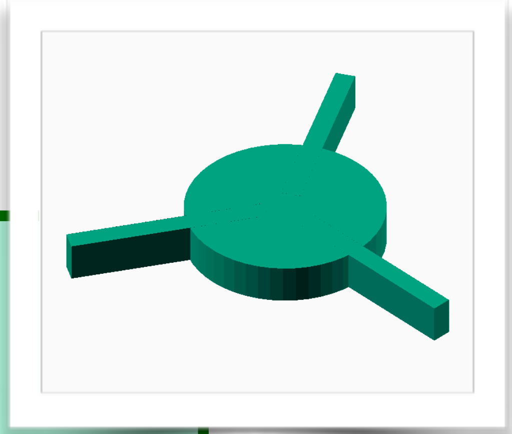
Szalinski

This talk



```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,–0.5,0]
     (Cuboid [10, 1, 1]))))))
```

edits

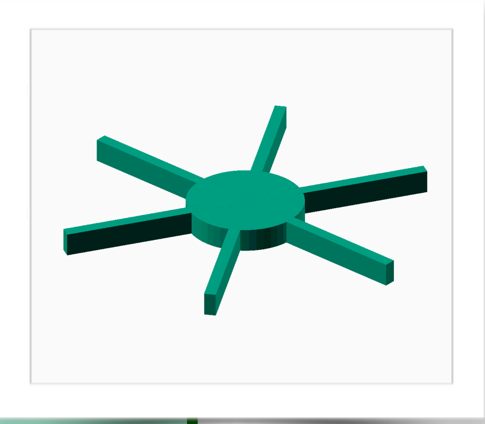


```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 3)
   (Rotate [0, 0, 120i]
    (Translate [1,–0.5,0]
     (Cuboid [10, 1, 1]))))))
```

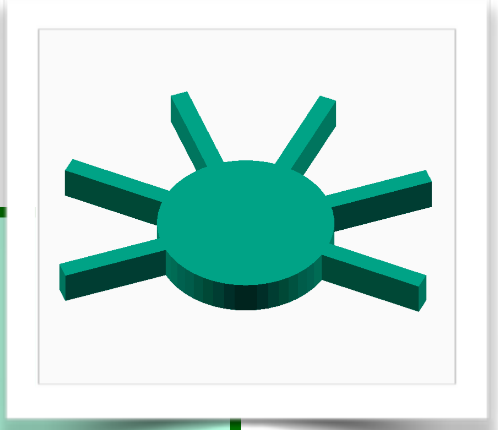
edits

Fold and Tabulate represent loops

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 8)
   (Rotate [0, 0, 45i]
    (Translate [1,–0.5,0]
     (Cuboid [15, 1, 1]))))))
```



```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,–0.5,0]
     (Cuboid [15, 1, 1]))))))
```



```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 45i]
    (Translate [1,–0.5,0]
     (Cuboid [10, 1, 1]))))))
```

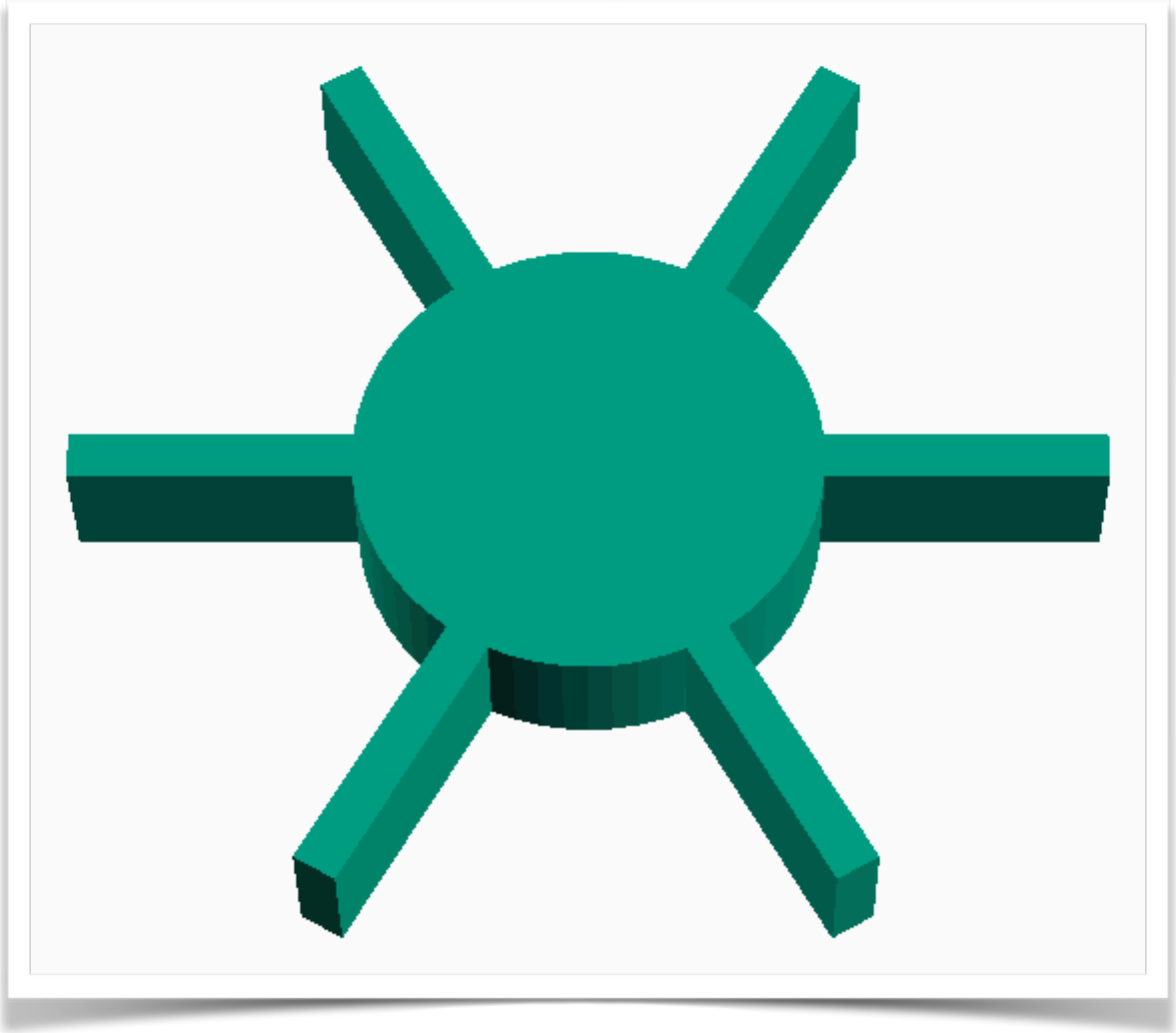
Szalinski: flat CAD ➔ parametrized CAD

Automatically infer loops from straight line programs in the form of **Folds**, **Maps**, and **Tabulates**

Hypothesis: Parametrized programs are easier to read/customize than flat programs

Ideal Input to Szalinski

```
(Union
 (Cylinder [1, 5])
 (Union
  (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))
```



Term Rewriting

```
(Union (Cylinder [1, 5])
 (Union
  (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))
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Term Rewriting

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(Union (Cylinder [1, 5])
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  (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
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  (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))
```

Fold Union Rewrite

```
(Union (Cylinder [1, 5, 5])
 (Fold Union (List
  (Rotate [0, 0, 0]
     (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
     (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
...
```

Term Rewriting

```
(Union (Cylinder [1, 5])
 (Union
  (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))
```

Fold Union Rewrite

```
(Union (Cylinder [1, 5, 5])
 (Fold Union (List
  (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
...
```

Structure Finder

```
(Union (Cylinder [1, 5, 5])
 (Fold Union
  (Map2 Rotate
   (List [0, 0, 0] [0, 0, 60] ... [0, 0, 300])
   (List
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])) ...
```

Term Rewriting

(Union (Cylinder [1, 5])
(Union
(Rotate [0, 0, 0]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 60]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 120]
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(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 240]
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(Rotate [0, 0, 300]
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→

Fold Union Rewrite

(Union (Cylinder [1, 5, 5])
(Fold Union (List
(Rotate [0, 0, 0]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 60]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
. . .

→

Structure Finder

(Union (Cylinder [1, 5, 5])
(Fold Union
(Map2 Rotate
(List [0, 0, 0] [0, 0, 60] … [0, 0, 300])
(List
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])) …

↓ *

Structure Finder

(Union (Cylinder [1, 5, 5])
(Fold Union
(Map2 Rotate
(List [0, 0, 0] [0, 0, 60] … [0, 0, 300])
(Map2 Translate
(**Repeat** 6 [1, −0.5, 0]
(**Repeat** 6 (Cuboid [10, 1, 1]))))))


Term Rewriting
(Union (Cylinder [1, 5])
(Union
(Rotate [0, 0, 0]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 60]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 120]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 180]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 240]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 300]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))
Fold Union Rewrite
(Union (Cylinder [1, 5, 5])
(Fold Union (List
(Rotate [0, 0, 0]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 60]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
...
Structure Finder
(Union (Cylinder [1, 5, 5])
(Fold Union
(Map2 Rotate
(List [0, 0, 0] [0, 0, 60] ... [0, 0, 300])
(List
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])) ...
*
Structure Finder
(Union (Cylinder [1, 5, 5])
(Fold Union
(Map2 Rotate
(List [0, 0, 0] [0, 0, 60] ... [0, 0, 300])
(Map2 Translate
(Repeat 6 [1, −0.5, 0]
(Repeat 6 (Cuboid [10, 1, 1]))))))
*
Custom Solver
(Union (Cylinder [1, 5, 5])
(Fold Union
(Map2 Rotate
(Tabulate (i 6) (0, 0, 60i))
(Map2 Translate
(Repeat 6 [1, −0.5, 0]
(Repeat 6 (Cuboid [10, 1, 1]))))))

Term Rewriting


Term Rewriting
(Union (Cylinder [1, 5])
(Union
(Rotate [0, 0, 0]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 60]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 120]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 180]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 240]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 300]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
Fold Union Rewrite
(Union (Cylinder [1, 5, 5])
(Fold Union (List
(Rotate [0, 0, 0]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 60]
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
. . .
Structure Finder
(Union (Cylinder [1, 5, 5])
(Fold Union
(Map2 Rotate
(List [0, 0, 0] [0, 0, 60] … [0, 0, 300])
(List
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))
(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])) …
*
Structure Finder
(Union (Cylinder [1, 5, 5])
(Fold Union
(Map2 Rotate
(List [0, 0, 0] [0, 0, 60] … [0, 0, 300])
(Map2 Translate
(Repeat 6 [1, −0.5, 0]
(Repeat 6 (Cuboid [10, 1, 1]))))))
*
Custom Solver
(Union (Cylinder [1, 5, 5])
(Fold Union
(Map2 Rotate
(Tabulate (i 6) (0, 0, 60i))
(Map2 Translate
(Repeat 6 [1, −0.5, 0]
(Repeat 6 (Cuboid [10, 1, 1]))))))
*
Lift Tabulate Rewrite
(Union (Cylinder [1, 5, 5])
(Fold Union
(Tabulate (i 6)
(Rotate [0, 0, 60i]
(Translate [1,−0.5,0]
(Cuboid [10, 1, 1]))))))

Structure Finder

```
List
  (Op [param 1] (arg 1))
  (Op [param 2] (arg 2))
  (Op [param 3] (arg 3)) …
```

→

```
Map2 Op
  (List [param 1] [param 2] [param 3])
  (List (arg 1) (arg 2) (arg 3))
```

Structure Finder

```
List
  (Op [param 1] (arg 1))
  (Op [param 2] (arg 2))
  (Op [param 3] (arg 3)) …
```

→

```
Map2 Op
  (List [param 1] [param 2] [param 3])
  (List (arg 1) (arg 2) (arg 3))
```

Fold Union Rewrite

```
(Union (Cylinder [1, 5])
 (Fold Union (List
 (Rotate [0, 0, 0]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 60]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 120]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 180]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 240]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 300]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))
```

→

```
(Union (Cylinder [1, 5, 5])
 (Fold Union
 (Map2 Rotate
   (List [0, 0, 0] [0, 0, 60] [0, 0, 120] [0, 0, 180] [0, 0, 240] [0, 0, 300])
   (List
     (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))
     (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])) …
```

List 1

List 2

Structure Finder

```
List
  (Op [param 1] (arg 1))
  (Op [param 2] (arg 2))
  (Op [param 3] (arg 3)) …
```

→

```
Map2 Op
  (List [param 1] [param 2] [param 3])
  (List (arg 1) (arg 2) (arg 3))
```

Fold Union Rewrite

```
(Union (Cylinder [1, 5])
 (Fold Union (List
 (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))
```

→

```
(Union (Cylinder [1, 5, 5])
 (Fold Union
 (Map2 Rotate
   (List [0, 0, 0] [0, 0, 60] [0, 0, 120] [0, 0, 180] [0, 0, 240] [0, 0, 300])
   (List
     (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))
     (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])) …
```

Map2 applies the operator to the i^{th} element of the first list and i^{th} element of the second list

List 1

List 2

Structure Finder

```
List
  (Op [param 1] (arg 1))
  (Op [param 2] (arg 2))
  (Op [param 3] (arg 3)) …
```

→

```
Map2 Op
  (List [param 1] [param 2] [param 3])
  (List (arg 1) (arg 2) (arg 3))
```

Fold Union Rewrite

```
(Union (Cylinder [1, 5])
 (Fold Union (List
 (Rotate [0, 0, 0]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 60]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 120]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 180]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 240]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
 (Rotate [0, 0, 300]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))))
```

→

```
(Union (Cylinder [1, 5, 5])
 (Fold Union
 (Map2 Rotate
   (List [0, 0, 0] [0, 0, 60] [0, 0, 120] [0, 0, 180] [0, 0, 240] [0, 0, 300])
   (List
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1]))
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])) …
```

Map2 applies the operator to the i^th element of the first list and i^th element of the second list

List 1

List 2

Also applies to this list

Custom Solvers

Structure Finder

```
(Union (Cylinder [1, 5, 5])
 (Fold Union
  (Map2 Rotate
   (List [0, 0, 0] [0, 0, 60] … [0, 0, 300])
   (Map2 Translate
    (Repeat 6 [1, −0.5, 0]
    (Repeat 6 (Cuboid [10, 1, 1]))))))
```

Custom solver

```
(Union (Cylinder [1, 5, 5])
 (Fold Union
  (Map2 Rotate
   (Tabulate (i 6) (0, 0, 60i))
   (Map2 Translate
    (Repeat 6 [1, −0.5, 0]
    (Repeat 6 (Cuboid [10, 1, 1]))))))
```

The concrete list of vectors is passed to a custom solver that finds a closed form arithmetic expression

Term Rewriting

Term Rewriting

Inputs to Szalinski are rarely ideal!

Ideal Input vs Actual Input

```
(Union
 (Cylinder [1, 5])
 (Union
  (Rotate [0, 0, 0]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))
```

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Union
    (Rotate [0,0,120]
      (Translate [1,−0.5,0] (Cuboid [10,1,1])))
    (Scale [10,1,1]
      (Translate [0.1,−0.5,1] (Cuboid [1,1,1])))
    (Rotate [0, 0, 300]
      (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
    (Translate [−1,0.5,0]
      (Scale [−1,−1,1] Cuboid [10,1,1]))
    (Rotate [0, 0, 240]
      (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
    (Rotate [0, 0, 60]
      (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))
```

Ideal Input vs Actual Input

```
(Union
 (Cylinder [1, 5])
 (Union
  (Rotate [0, 0, 0]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))))
```

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Union
    (Rotate [0,0,120]
      (Translate [1,–0.5,0] (Cuboid [10,1,1])))
    (Scale [10,1,1]
      (Translate [0.1,–0.5,1] (Cuboid [1,1,1])))
    (Rotate [0, 0, 300]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
    (Translate [–1,0.5,0]
      (Scale [–1,–1,1] Cuboid [10,1,1]))
    (Rotate [0, 0, 240]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
    (Rotate [0, 0, 60]
      (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))))
```

Ideal Input vs Actual Input

Previous rewriting strategy no longer works!

```
(Rotate [0, 0, 0]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 60]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 120]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 180]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 240]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 300]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))))
```

```
(Rotate [0,0,120]
  (Translate [1,–0.5,0] (Cuboid [10,1,1])))
(Scale [10,1,1]
  (Translate [0.1,–0.5,1] (Cuboid [1,1,1])))
(Rotate [0, 0, 300]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
(Translate [–1,0.5,0]
  (Scale [–1,–1,1] Cuboid [10,1,1]))
(Rotate [0, 0, 240]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 60]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))))
```

Ideal Input vs Actual Input

Must <u>interleave</u> rewriting strategy with CAD identities to line up subexpressions

```
(Rotate [0, 0, 60]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 120]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 180]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 240]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 300]
  (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))))
```

```
  (Scale [10,1,1]
    (Translate [0.1,–0.5,1] (Cuboid [1,1,1])))
  (Rotate [0, 0, 300]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Translate [–1,0.5,0]
    (Scale [–1,–1,1] Cuboid [10,1,1]))
  (Rotate [0, 0, 240]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))))
```

Ideal Input vs Actual Input

Must <u>interleave</u> rewriting strategy with CAD identities to line up subexpressions

Phase ordering problem: order of rewriting matters!

(Rotate [0, 0, 60]

(Rotate [0, 0, 180]
  (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 240]
  (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 300]
  (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))

(Scale [10,1,1]

(Translate [−1,0.5,0]
  (Scale [−1,−1,1] Cuboid [10,1,1]))
(Rotate [0, 0, 240]
  (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
(Rotate [0, 0, 60]
  (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))

Ideal Input vs Actual Input

Must <u>interleave</u> rewriting strategy with CAD identities to line up subexpressions

(Rotate [0, 0, 60]

(Scale [10,1,1]

Phase ordering problem: order of rewriting matters!

(Rotate [0, 0, 180]

(Translate [−1,0.5,0]

E-graphs* can solve phase ordering

(Rotate [0, 0, 300]

(Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))

(Rotate [0, 0, 60]

(Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))

* Equality Saturation: A New Approach to Optimization. Tate, Stepp, Tatlock, Lerner. POPL'09

Semantically Equivalent, Syntactically Different

```
(Union
 (Cylinder [1, 5])
 (Fold Union (List
  (Translate [1, −0.5, 0] (Cube [10, 1, 1]))
  (Rotate [0,0,60]
   (Translate [1,−0.5,0] (Cube[10,1,1])))
  (Rotate [0,0,120]
   (Translate [1,−0.5,0] (Cube[10,1,1])))
  (Scale  [−1,−1,1]
   (Translate [1,−0.5,0] (Cube[10,1,1])))
  (Rotate [0,0,240]
   (Translate [1,−0.5,0] (Cube[10,1,1])))
  (Rotate [0, 0, 300]
   (Translate [1, −0.5, 0] (Cube [10, 1, 1]))))))
```

Rotate [0, 0, 180] is replaced by equivalent Scale [-1, -1, 1]

Semantically Equivalent, Syntactically Different

Rotate [0, 0, 180]
  (Translate [1,–0.5,0] (Cube[10,1,1]))

=

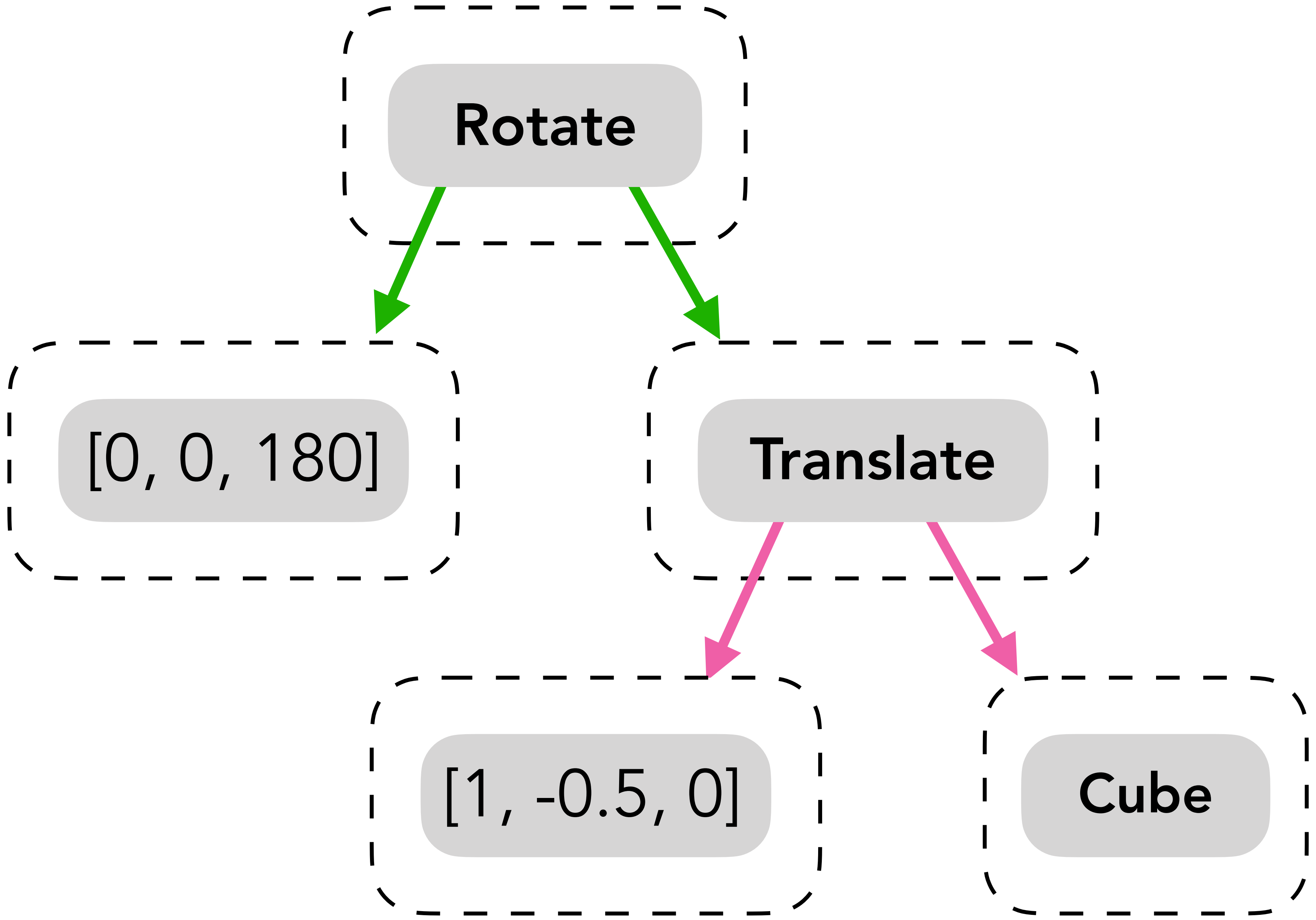
Scale [-1, -1, 1]
  (Translate [1,–0.5,0] (Cube[10,1,1]))

Syntactic rewrite

$$Rotate\ (0,\ 0,\ 180,\ c)) \leftrightsquigarrow Scale\ (-1,\ -1,\ 1,\ c))$$

Store Expressions in an E-graph

Rotate [0, 0, 180]
(Translate [1,–0.5,0] (Cube[10,1,1])) = Scale [-1, -1, 1]
(Translate [1,–0.5,0] (Cube[10,1,1]))

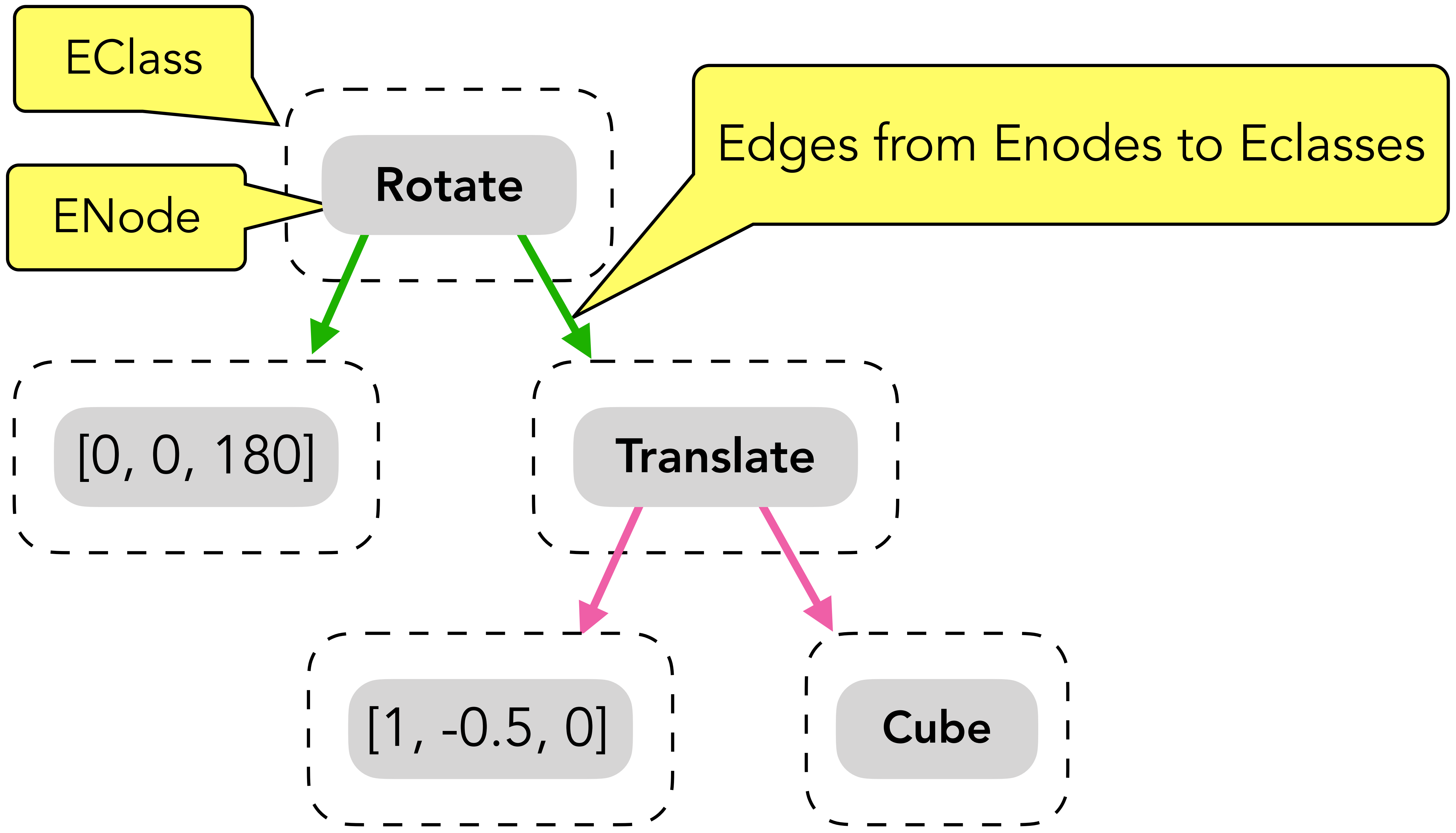


Syntactic rewrite

$$\mathit{Rotate}\ (0,\ 0,\ 180,\ c)) \rightsquigarrow \mathit{Scale}\ (-1,\ -1,\ 1,\ c))$$

Store Expressions in an E-graph

Rotate [0, 0, 180] (Translate [1,–0.5,0] (Cube[10,1,1])) = Scale [-1, -1, 1] (Translate [1,–0.5,0] (Cube[10,1,1]))

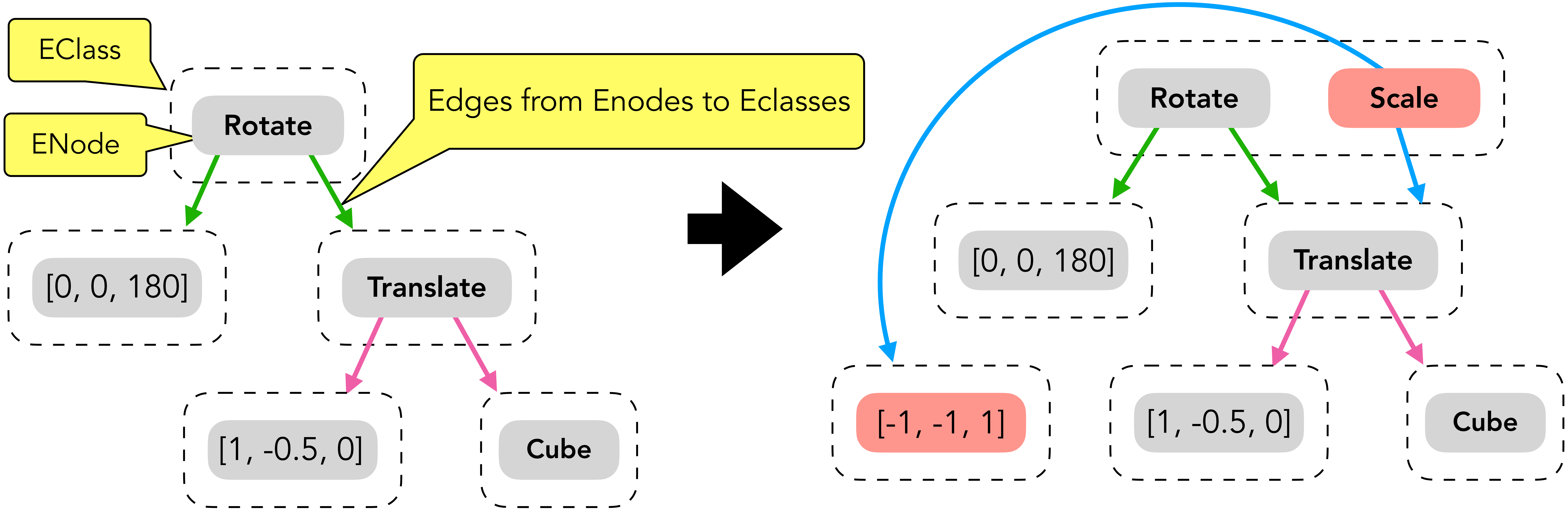


Syntactic rewrite

$$\mathit{Rotate}\ (0,\ 0,\ 180,\ c)) \leftrightsquigarrow \mathit{Scale}\ (-1,\ -1,\ 1,\ c))$$

Store Expressions in an E-graph

Rotate [0, 0, 180]
(Translate [1,–0.5,0] (Cube[10,1,1])) = Scale [-1, -1, 1]
(Translate [1,–0.5,0] (Cube[10,1,1]))



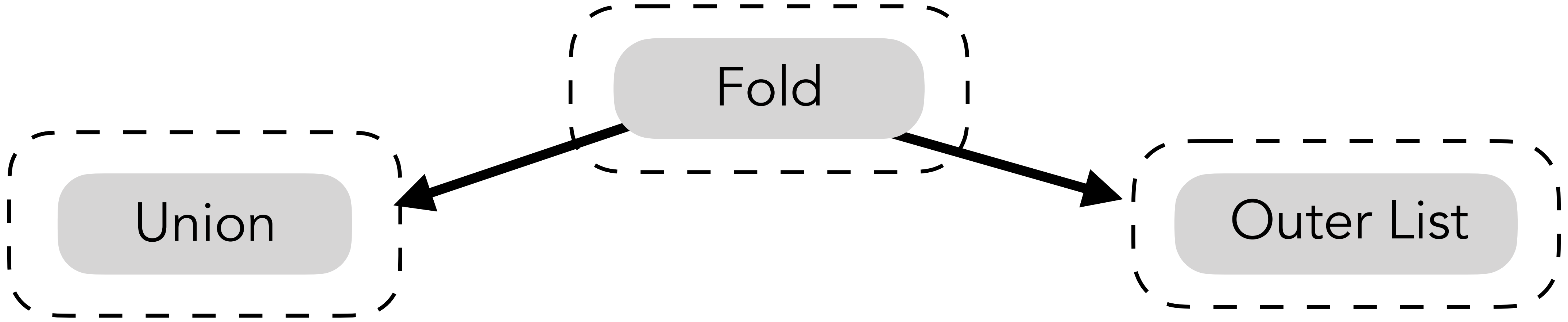
Syntactic rewrite

$$Rotate\ (0,\ 0,\ 180,\ c)) \leftrightsquigarrow Scale\ (-1,\ -1,\ 1,\ c))$$

Custom Solvers in E-graph

```
(Union
 (Cylinder [1, 5])
 (Fold Union (List
  (Rotate [0, 0, 0]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```

Outer list



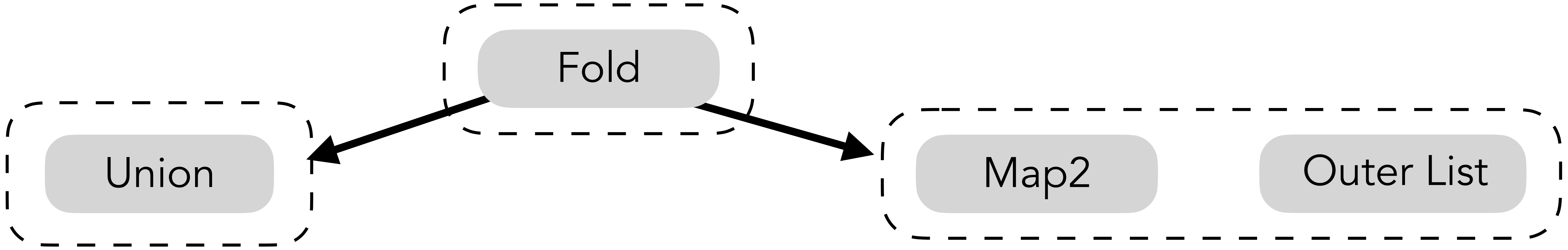
Custom Solvers in E-graph

```
(Union
 (Cylinder [1, 5])
 (Fold Union (List
  (Rotate [0, 0, 0]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))))
```

Outer list

Structure Finder

```
(Union (Cylinder [1, 5, 5])
 (Fold Union
  (Map2 Rotate
   (List [0, 0, 0] [0, 0, 60] … [0, 0, 300])
   (List
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])) …
```



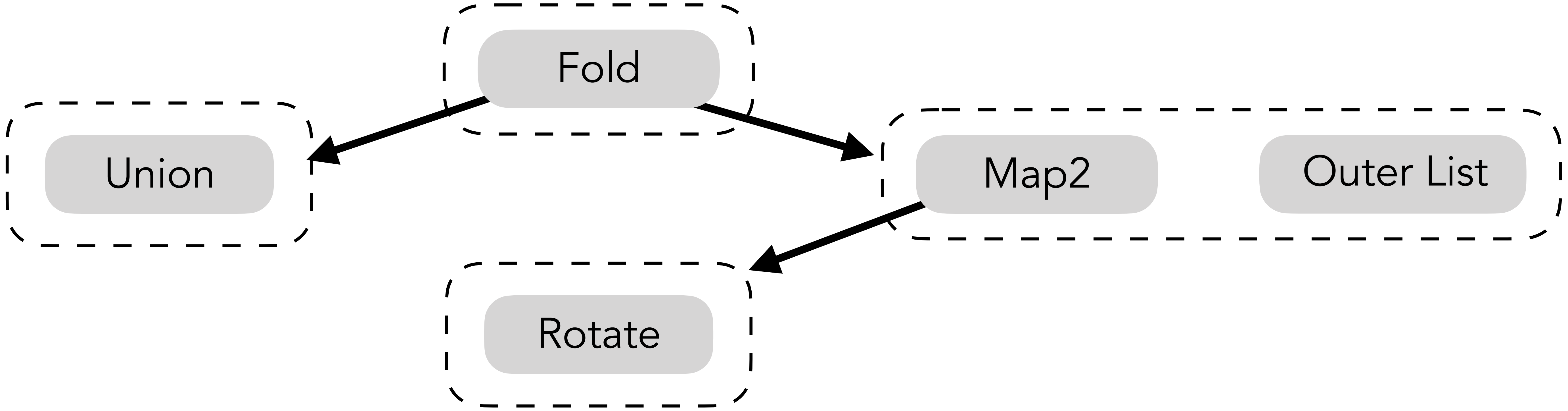
Custom Solvers in E-graph

```
(Union
 (Cylinder [1, 5])
 (Fold Union (List
  (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```

Outer list

Structure Finder

```
(Union (Cylinder [1, 5, 5])
 (Fold Union
  (Map2 Rotate
   (List [0, 0, 0] [0, 0, 60] … [0, 0, 300])
   (List
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])) …
```



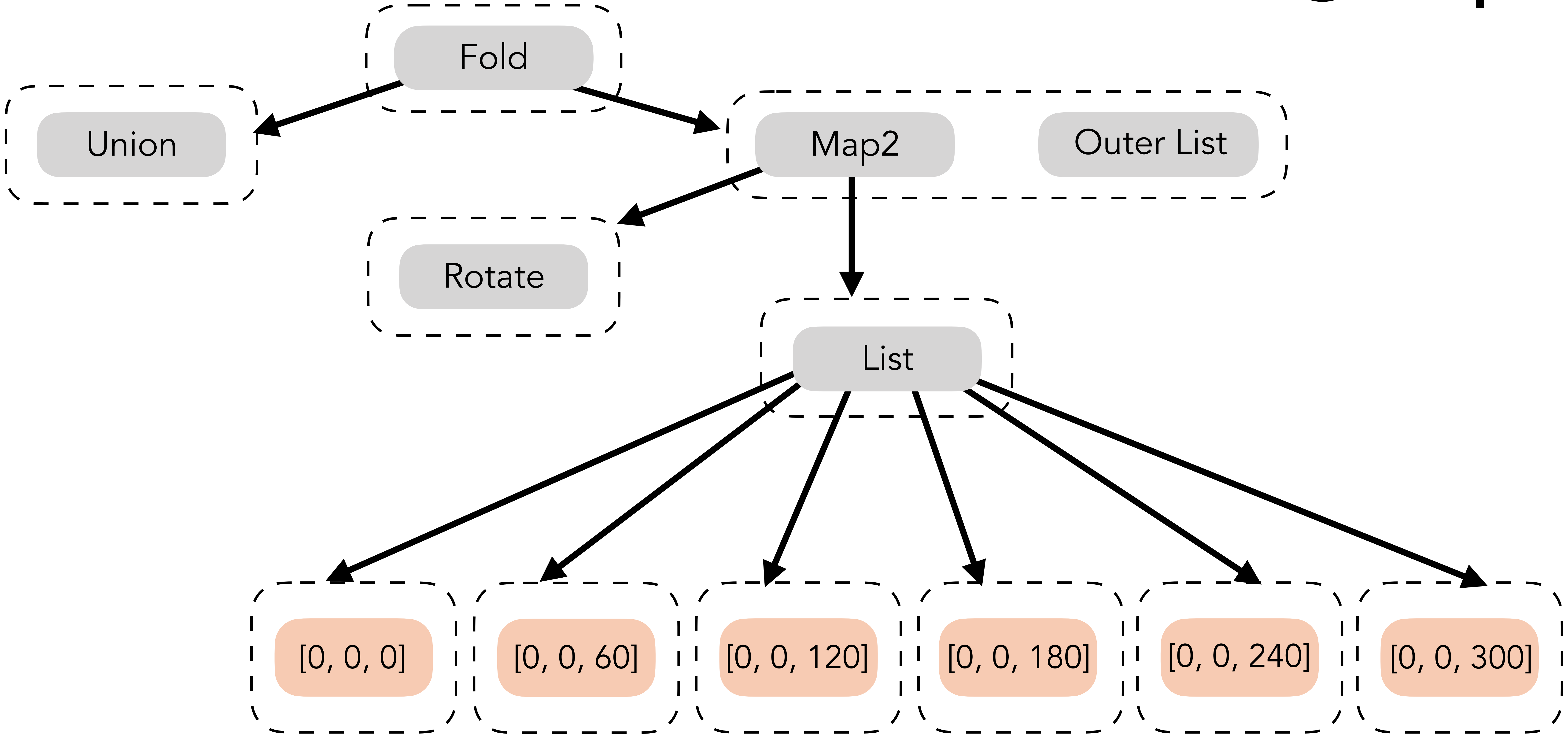
Custom Solvers in E-graph

```
(Union
 (Cylinder [1, 5])
 (Fold Union (List
  (Rotate [0, 0, 0]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))))
```

Outer list

Structure Finder

```
(Union (Cylinder [1, 5, 5])
 (Fold Union
  (Map2 Rotate
   (List [0, 0, 0] [0, 0, 60] … [0, 0, 300])
   (List
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])) …
```



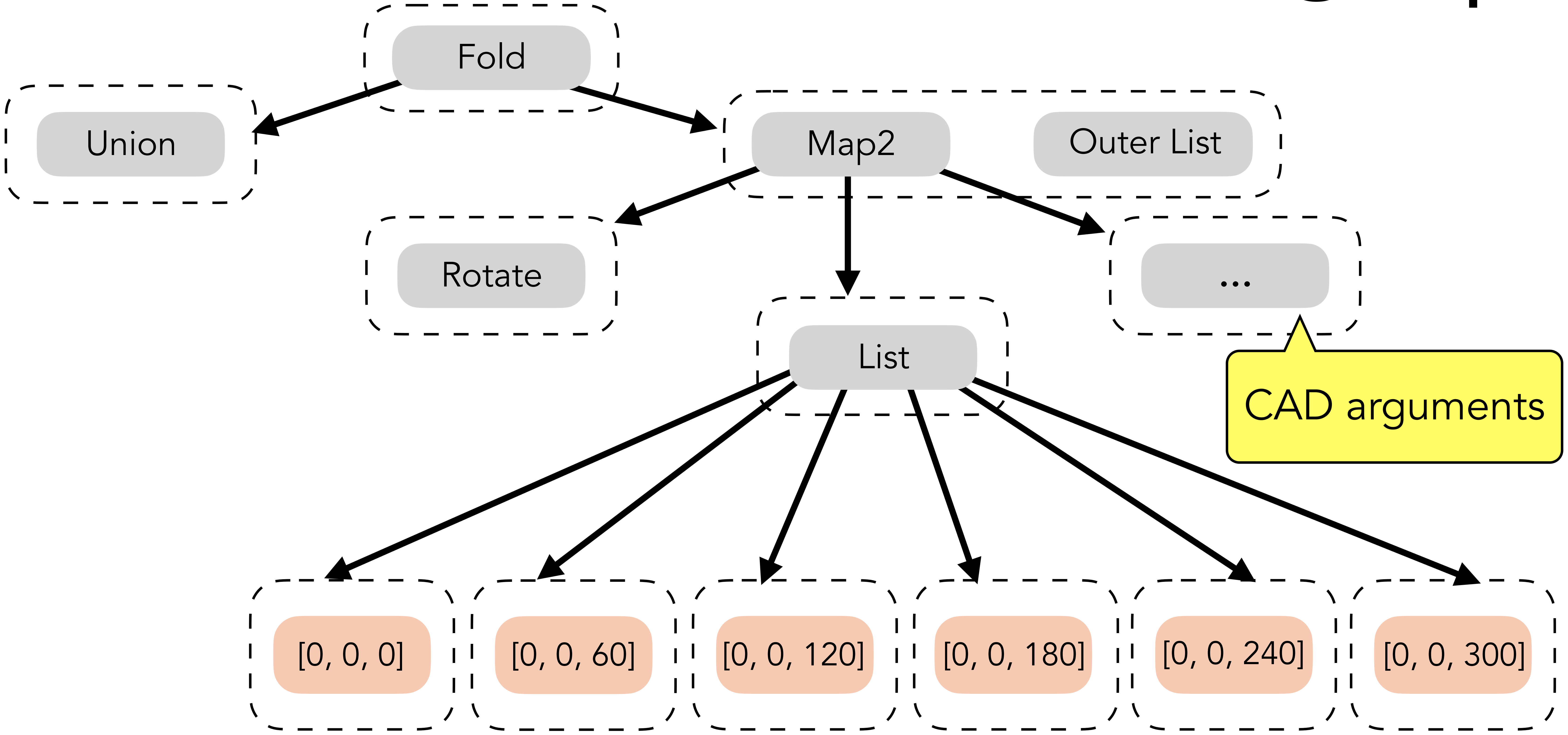
Custom Solvers in E-graph

```
(Union
 (Cylinder [1, 5])
 (Fold Union (List
  (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```

Outer list

Structure Finder

```
(Union (Cylinder [1, 5, 5])
 (Fold Union
  (Map2 Rotate
   (List [0, 0, 0] [0, 0, 60] … [0, 0, 300])
   (List
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])) …
```



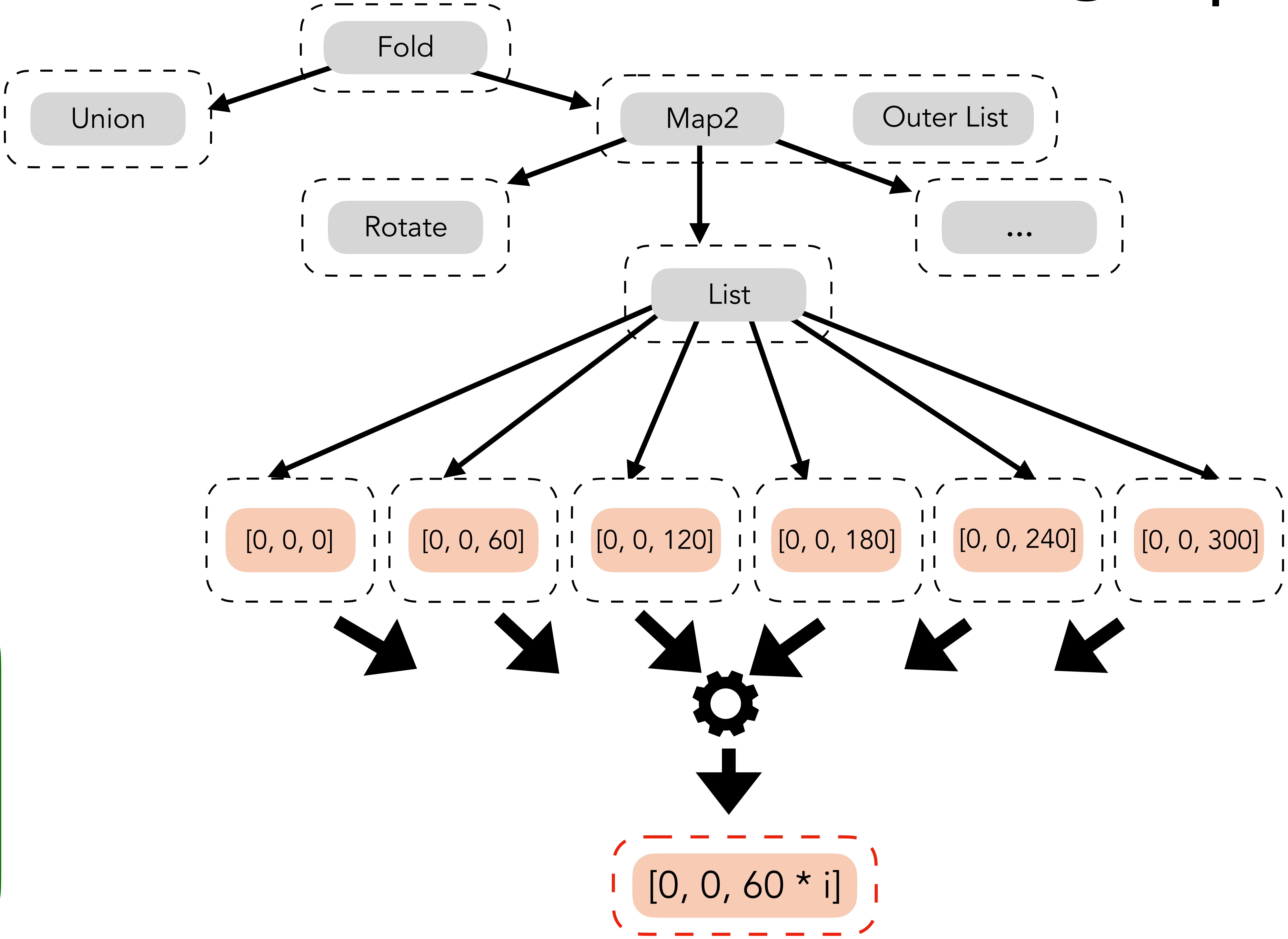
Custom Solvers in E-graph

```
(Union
 (Cylinder [1, 5])
 (Fold Union (List
  (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```

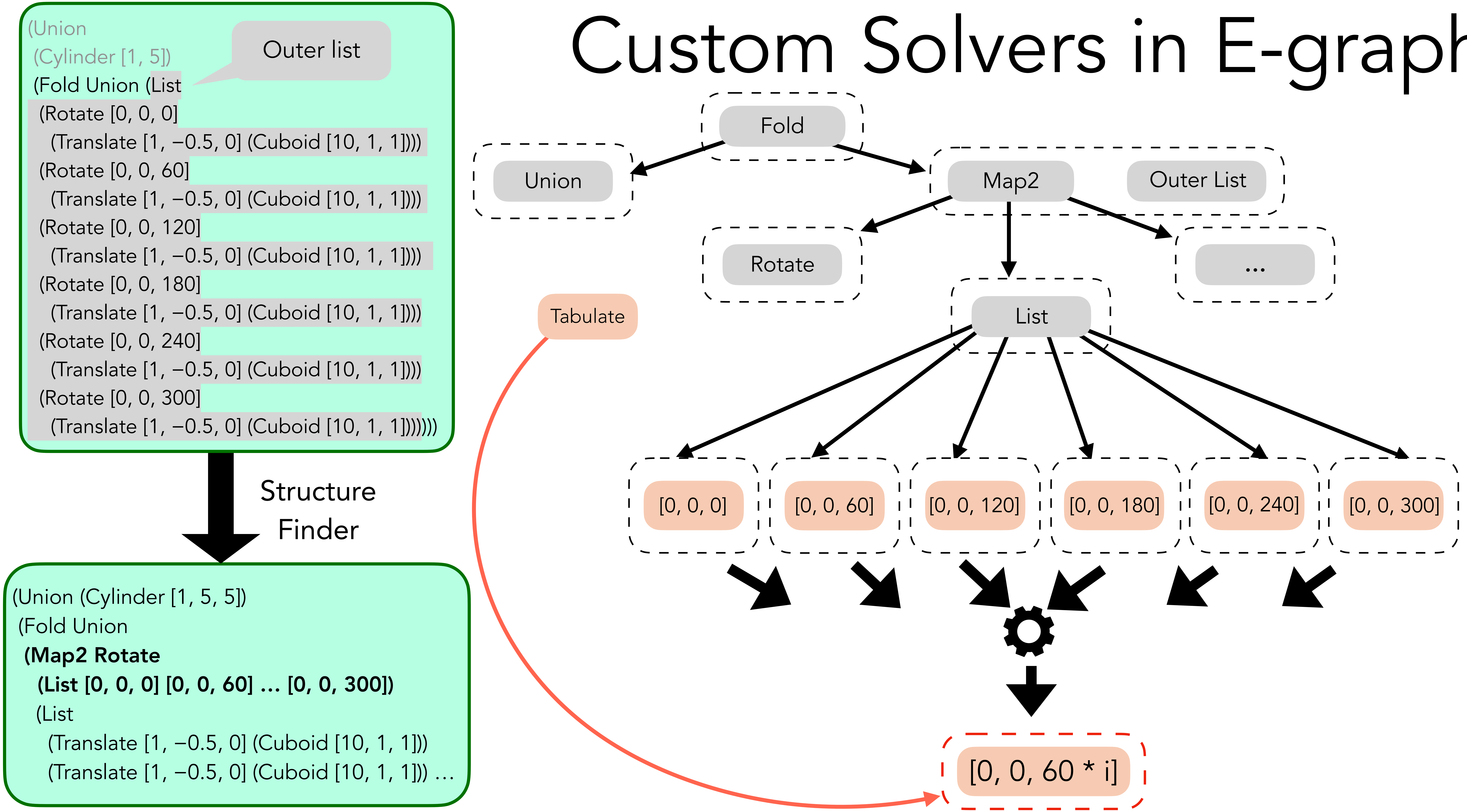
Outer list

Structure Finder

```
(Union (Cylinder [1, 5, 5])
 (Fold Union
  (Map2 Rotate
   (List [0, 0, 0] [0, 0, 60] … [0, 0, 300])
   (List
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])) …
```



Custom Solvers in E-graph



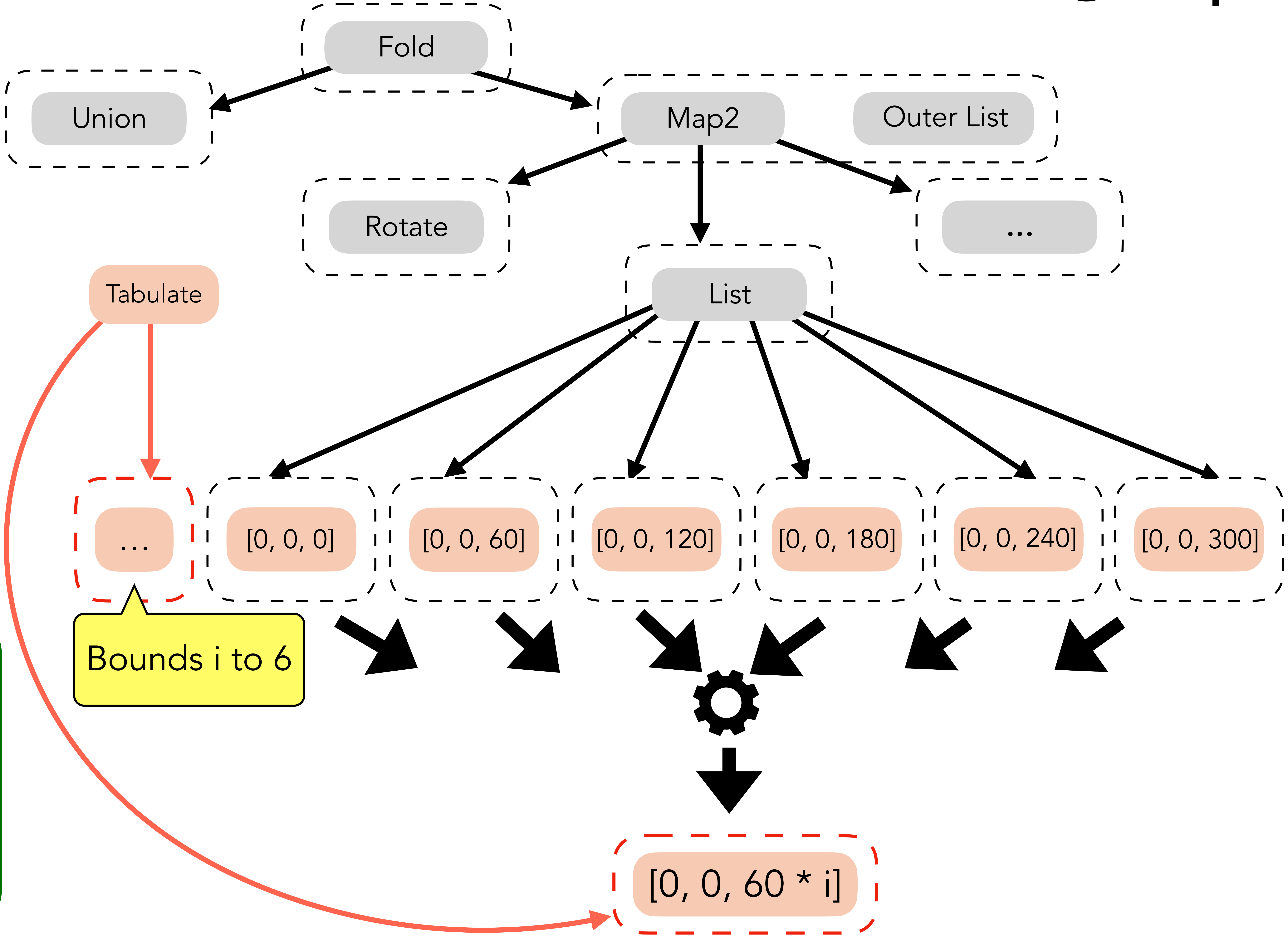
Custom Solvers in E-graph

```
(Union
 (Cylinder [1, 5])
 (Fold Union (List
  (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))))))
```

Outer list

Structure Finder

```
(Union (Cylinder [1, 5, 5])
 (Fold Union
  (Map2 Rotate
   (List [0, 0, 0] [0, 0, 60] … [0, 0, 300])
   (List
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])) …
```



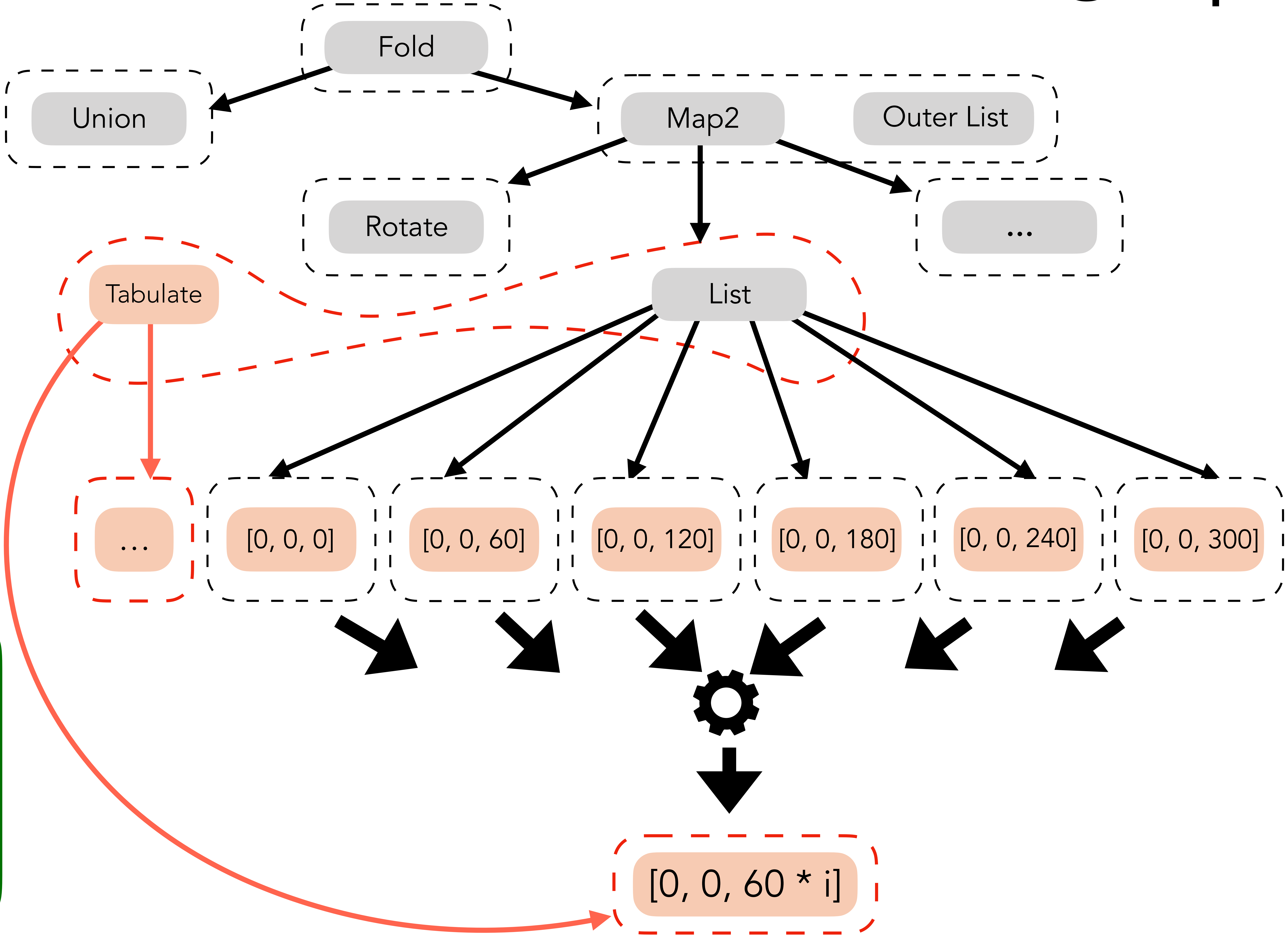
Custom Solvers in E-graph

```
(Union
 (Cylinder [1, 5])
 (Fold Union (List
  (Rotate [0, 0, 0]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 120]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```

Outer list

Structure Finder

```
(Union (Cylinder [1, 5, 5])
 (Fold Union
  (Map2 Rotate
   (List [0, 0, 0] [0, 0, 60] … [0, 0, 300])
   (List
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])) …
```



Custom Solvers for Non-Ideal Inputs

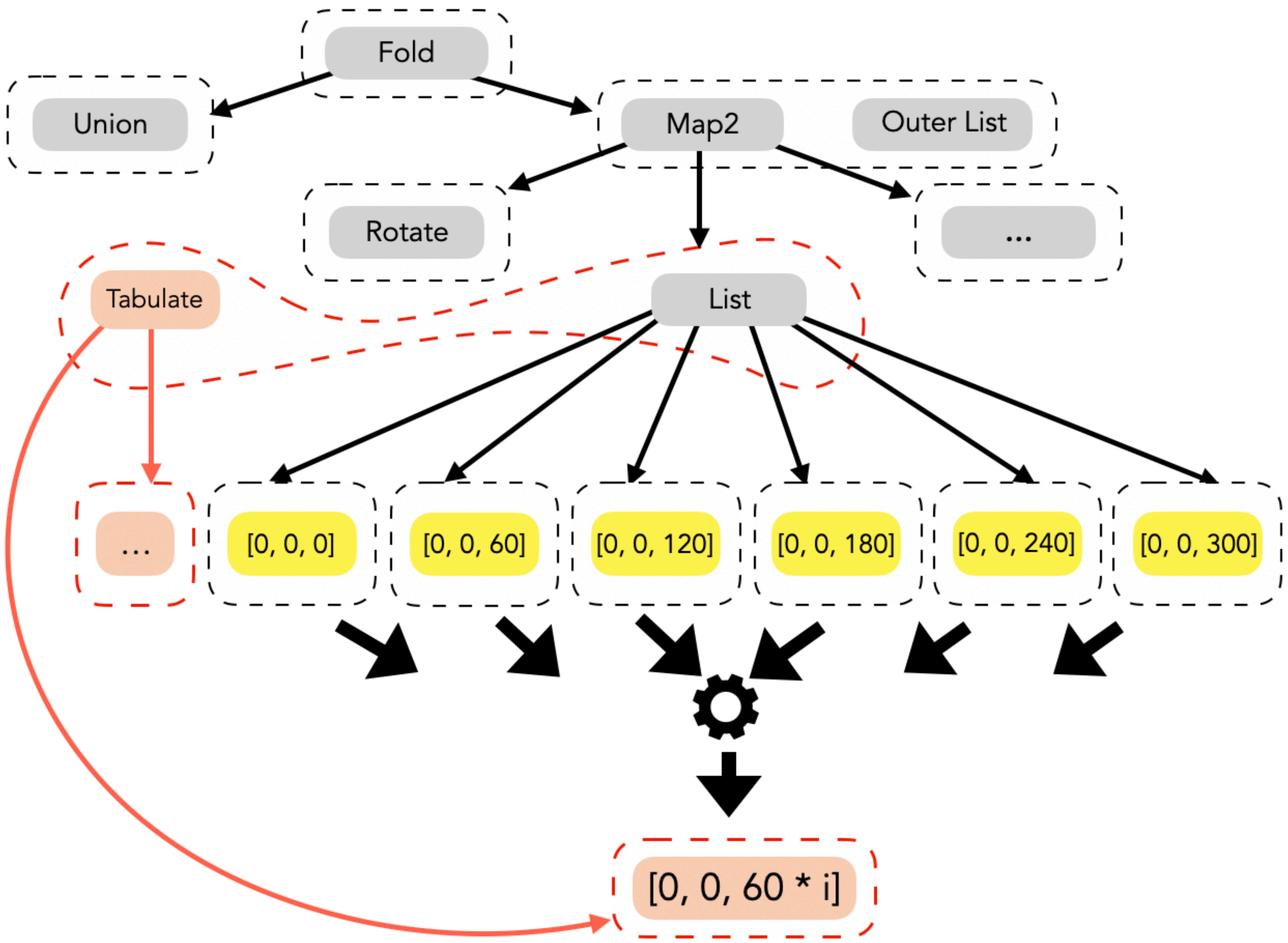
```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Fold Union (List
   (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```

Expressions are arbitrarily ordered

Parameters of Rotate are not sorted

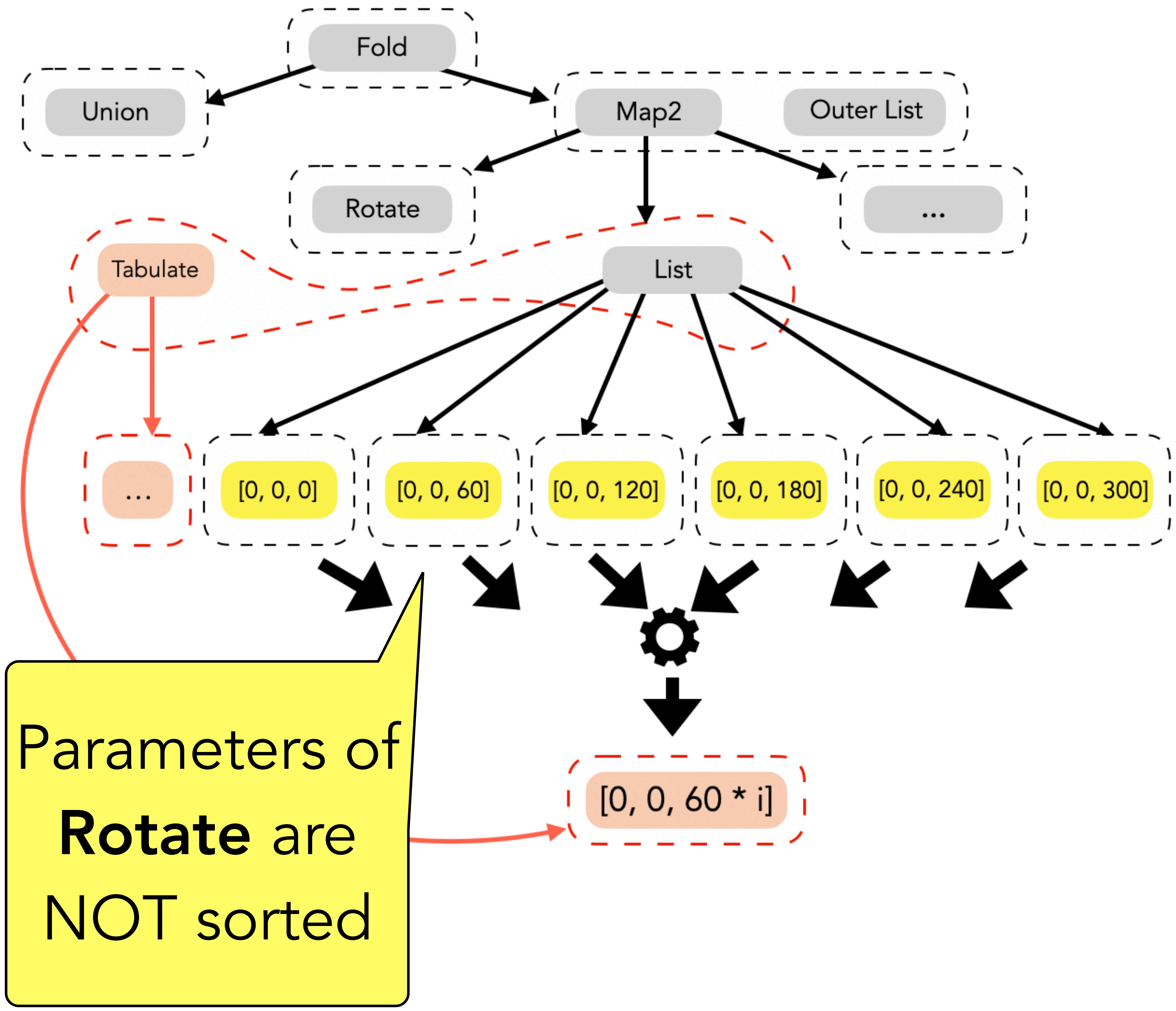
Custom Solvers for Non-Ideal Inputs

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Fold Union (List
   (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```



Custom Solvers for Non-Ideal Inputs

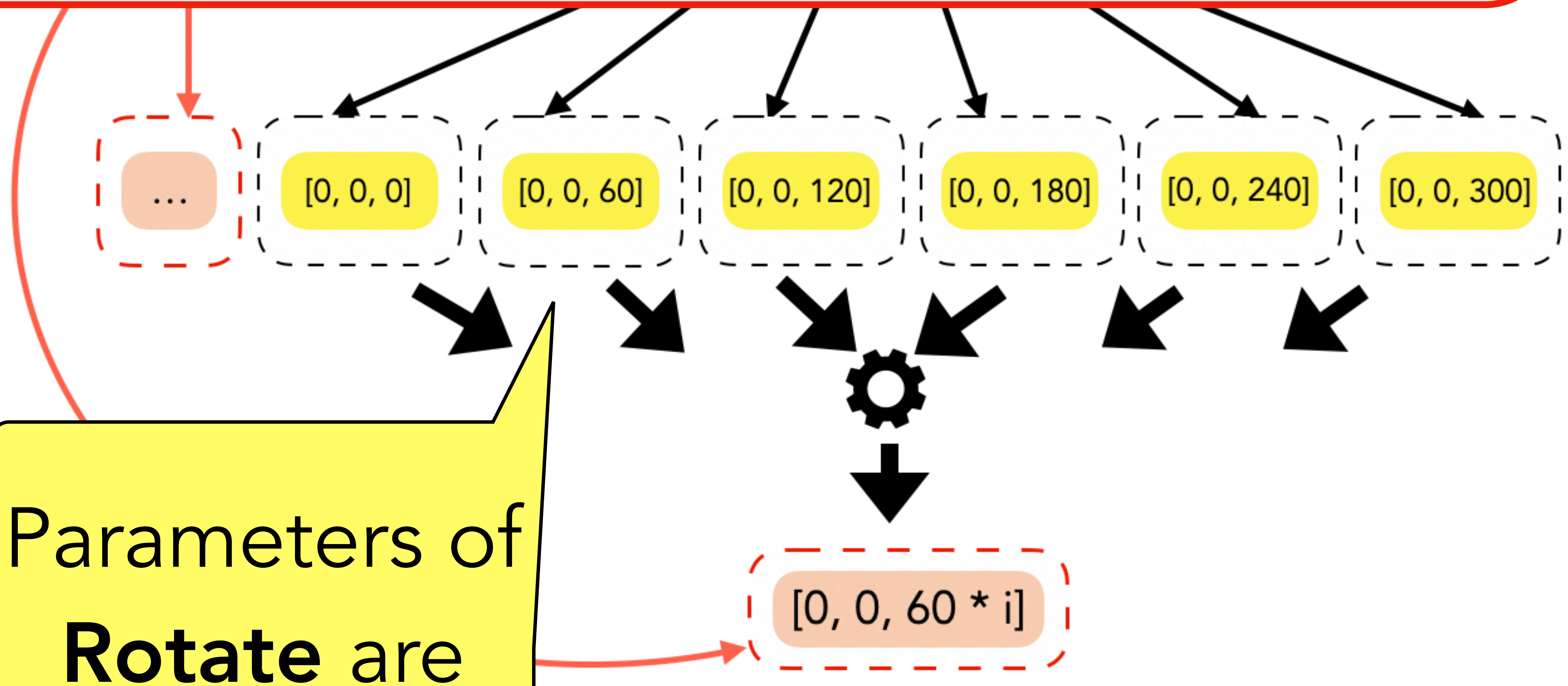
```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Fold Union (List
   (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```



Custom Solvers for Non-Ideal Inputs

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Fold Union (List
   (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```

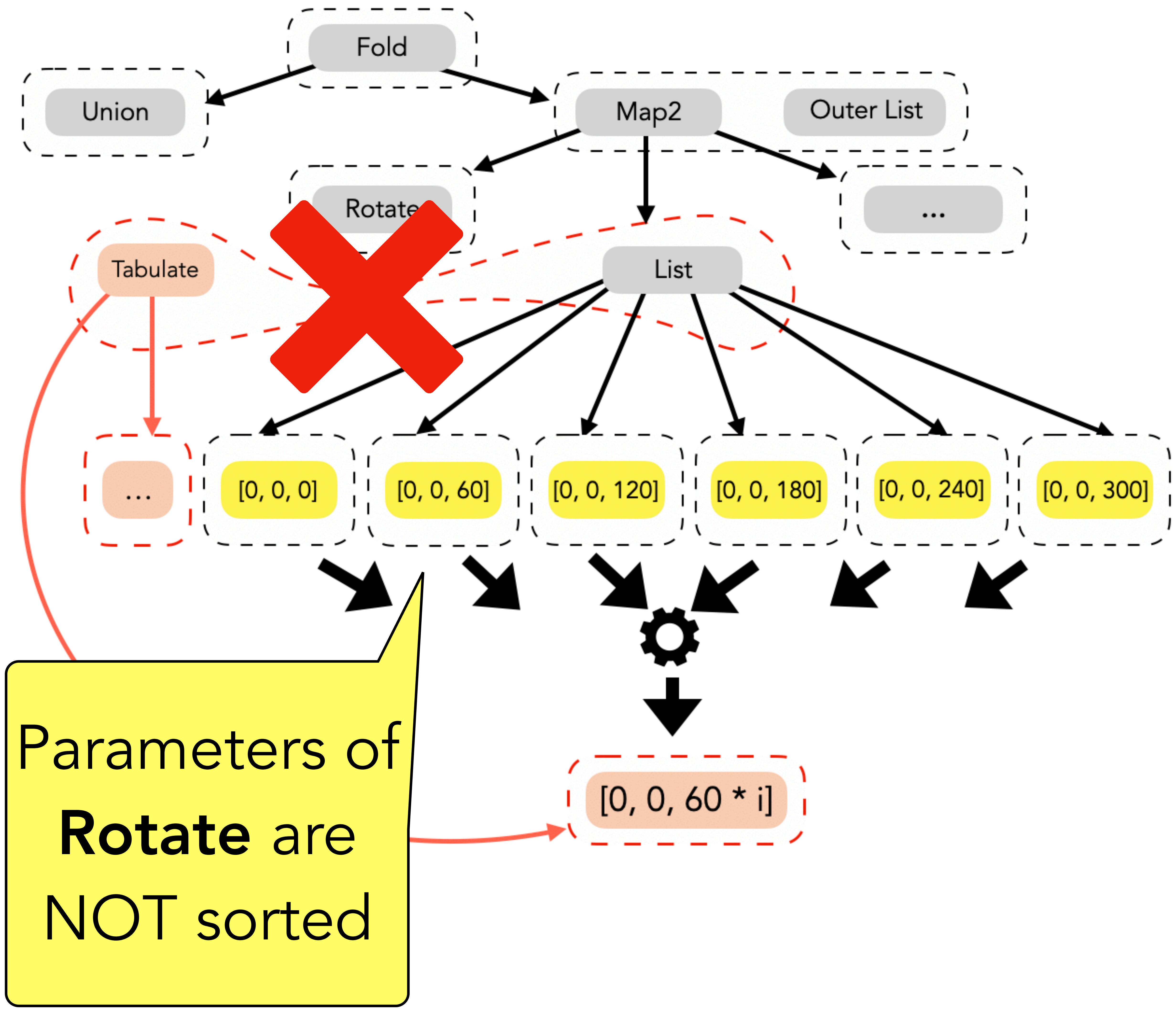
List of vectors must be sorted for the solver to be able to find the closed form and unify the Tabulate with the concrete list



Parameters of **Rotate** are NOT sorted

Custom Solvers for Non-Ideal Inputs

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Fold Union (List
   (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```



Naive Solution for Finding Closed Form

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Fold Union (List
   (Rotate [0, 0, 120]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 0]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 300]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 180]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 240]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 60]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1]))))))
```

Add all permutations of the list elements in the E-graph

Naive Solution Causes the AC-Matching Problem

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Fold Union (List
   (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```

Add all permutations of the list elements in the E-graph

Exponentially many choices in an E-graph due to associative-commutative operations like permuting lists, called AC-matching in the SMT community

Inverse Transformations

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Fold Union (List
   (Rotate [0, 0, 120]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 0]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 300]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 180]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 240]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 60]
    (Translate [1, –0.5, 0] (Cuboid [10, 1, 1]))))))
```

Key insight: allows solvers to speculatively transform their inputs to enable more profitable rewriting

Inverse Transformations

```
(Union
 (Scale [5,5,1] (Cylinder [1,1]))
 (Fold Union (List
  (Rotate [0, 0, 120]
   (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 0]
   (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
   (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
   (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
   (Translate [1, –0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
   (Translate [1, –0.5, 0] (Cuboid [10, 1, 1]))))))
```

Goal

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,–0.5,0]
     (Cuboid [10, 1, 1]))))))
```

Inverse Transformations

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Fold Union (List
   (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```

→ Structure Finder

```
(Fold Union
 (Map2 Rotate
  (List [0, 0, 120] [0, 0, 0] [0, 0, 300] [0, 0, 180] [0, 0, 240] [0, 0, 60])
  (Repeat 6
   (Translate [1, −0.5, 0]
    (Cuboid [10, 1, 1])))))
```

Goal

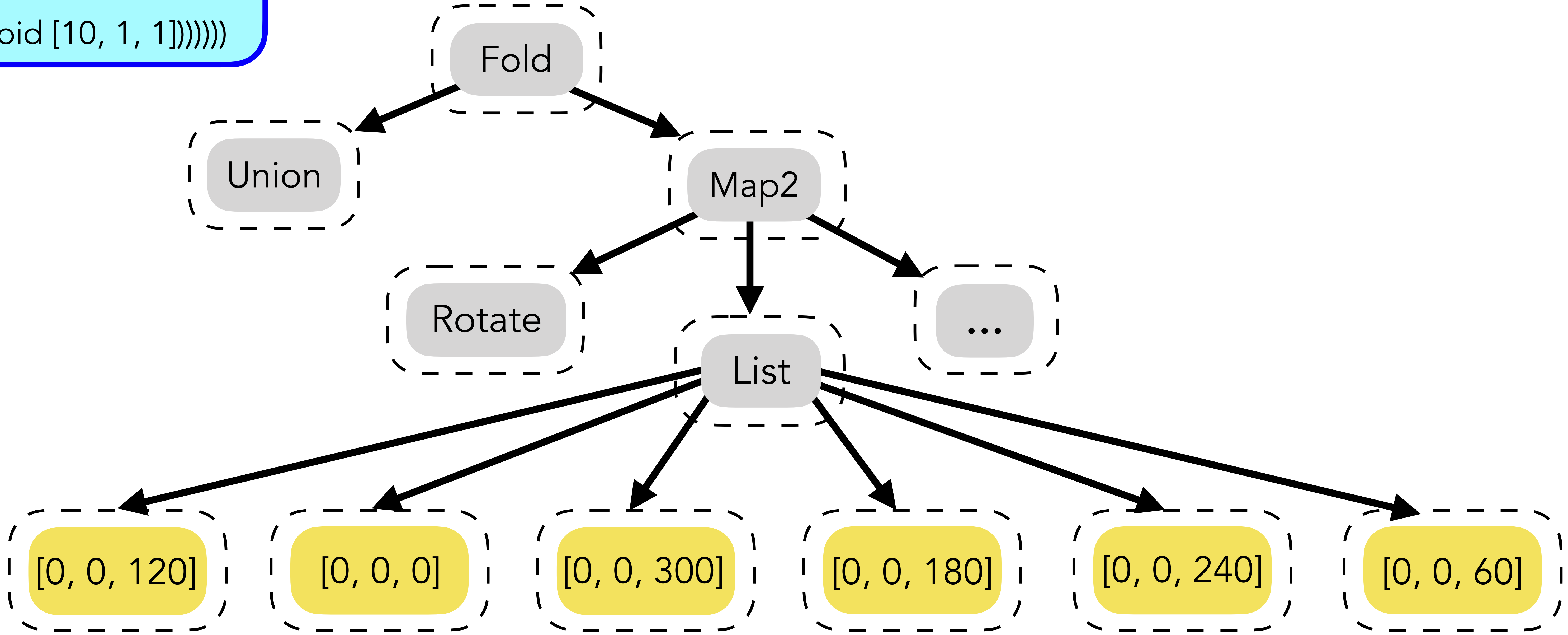
```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,−0.5,0]
     (Cuboid [10, 1, 1]))))))
```

Inverse Transformations

```
(Union
 (Scale [5,5,1] (Cylinder [1,1]))
 (Fold Union (List
  (Rotate [0, 0, 120]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 0]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 300]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 180]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 240]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
  (Rotate [0, 0, 60]
   (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```

Structure Finder

```
(Fold Union
 (Map2 Rotate
  (List [0, 0, 120] [0, 0, 0] [0, 0, 300] [0, 0, 180] [0, 0, 240] [0, 0, 60])
  (Repeat 6
   (Translate [1, −0.5, 0]
    (Cuboid [10, 1, 1])))))
```



Goal

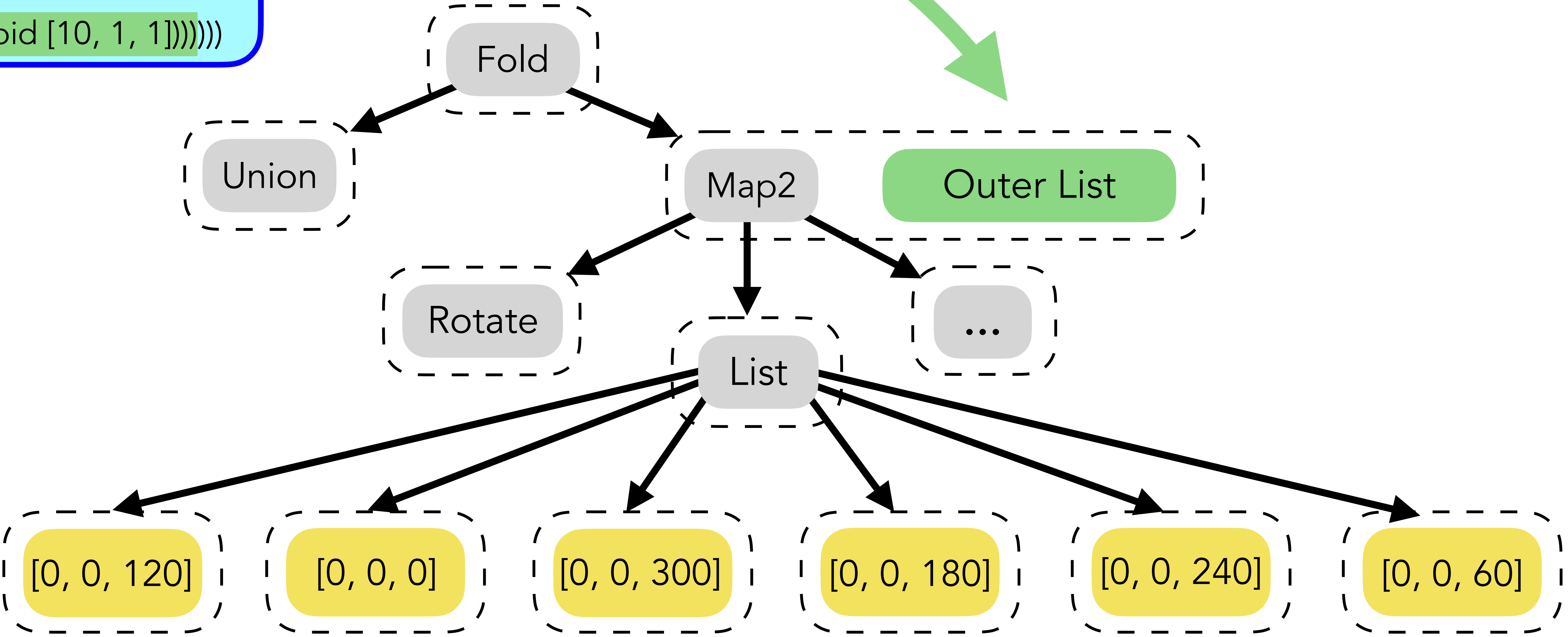
```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,−0.5,0]
     (Cuboid [10, 1, 1]))))))
```

Inverse Transformations

```
(Union
  (Scale [5,5,1] (Cylinder [1,1]))
  (Fold Union (List
   (Rotate [0, 0, 120]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 0]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 300]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 180]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 240]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1])))
   (Rotate [0, 0, 60]
    (Translate [1, −0.5, 0] (Cuboid [10, 1, 1]))))))
```

Structure Finder

```
(Fold Union
 (Map2 Rotate
  (List [0, 0, 120] [0, 0, 0] [0, 0, 300] [0, 0, 180] [0, 0, 240] [0, 0, 60])
  (Repeat 6
   (Translate [1, −0.5, 0]
    (Cuboid [10, 1, 1])))))
```



Goal

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,−0.5,0]
     (Cuboid [10, 1, 1]))))))
```

Inverse Transformations

```
(Fold Union
 (Map2 Rotate
  (List [0, 0, 120] [0, 0, 0] [0, 0, 300] [0, 0, 180] [0, 0, 240] [0, 0, 60])
  (Repeat 6
   (Translate [1, −0.5, 0]
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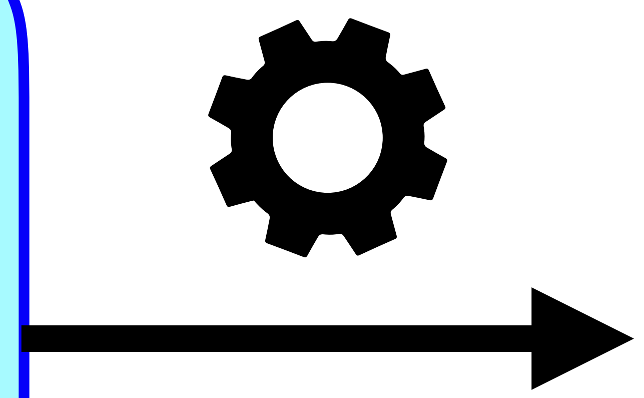

Goal

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,−0.5,0]
     (Cuboid [10, 1, 1]))))))
```

Inverse Transformations

```
(Fold Union
 (Map2 Rotate
  (List [0, 0, 120] [0, 0, 0] [0, 0, 300] [0, 0, 180] [0, 0, 240] [0, 0, 60])
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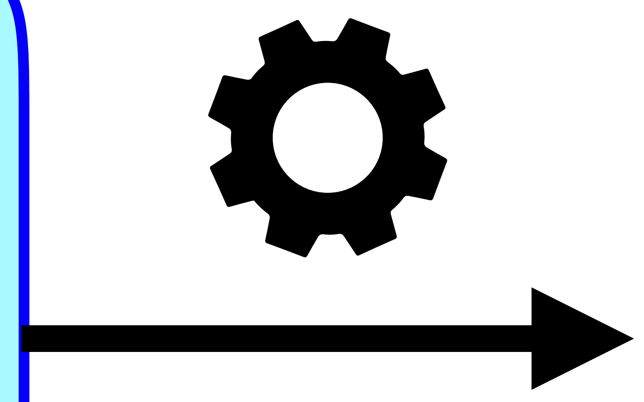

Solver permutes the list to find closed form!

Goal

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,−0.5,0]
     (Cuboid [10, 1, 1]))))))
```

Inverse Transformations

```
(Fold Union
 (Map2 Rotate
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   (Repeat 6
     (Translate [1, −0.5, 0]
       (Cuboid [10, 1, 1])))))
```

Solver permutes the list to find closed form!

```
(Fold Union
 (Map2 Rotate
   (Unsort <1 5 0 3 4 2> (Tabulate (i 6) [0, 0, 60 * i]))
   (Repeat 6
     (Translate [1, −0.5, 0]
       (Cuboid [10, 1, 1])))))
```

Goal

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
   (Tabulate (i 6)
     (Rotate [0, 0, 60i]
       (Translate [1,−0.5,0]
         (Cuboid [10, 1, 1]))))))
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Inverse Transformations

```
(Fold Union
 (Map2 Rotate
  (List [0, 0, 120] [0, 0, 0] [0, 0, 300] [0, 0, 180] [0, 0, 240] [0, 0, 60])
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Solver permutes the list to find closed form!

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(Fold Union
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  (Unsort <1 5 0 3 4 2> (Tabulate (i 6) [0, 0, 60 * i]))
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   (Translate [1, −0.5, 0]
    (Cuboid [10, 1, 1])))))
```

Solver annotates the expression with the profitable permutation

Goal

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,−0.5,0]
     (Cuboid [10, 1, 1]))))))
```

Inverse Transformations

```
(Fold Union
 (Map2 Rotate
  (List [0, 0, 120] [0, 0, 0] [0, 0, 300] [0, 0, 180] [0, 0, 240] [0, 0, 60])
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```
(Fold Union
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   (Translate [1, −0.5, 0]
    (Cuboid [10, 1, 1])))))
```

Solver annotates the expression with the profitable permutation

If a solver cannot simplify A, but it can simplify f(A) to B, then f^{-1}(B) can be unified with A

Inverse Transformations

```
(Fold Union
 (Map2 Rotate
   (List [0, 0, 120] [0, 0, 0] [0, 0, 300] [0, 0, 180] [0, 0, 240] [0, 0, 60])
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(Fold Union
 (Map2 Rotate
   (Unsort <1 5 0 3 4 2> (Tabulate (i 6) [0, 0, 60 * i]))
   (Repeat 6
     (Translate [1, −0.5, 0]
       (Cuboid [10, 1, 1])))))
```

Solver annotates the expression with the profitable permutation

Flexibly combines solvers with an egraph-driven rewrite system

Solvers allowed to transform their input however they want

BUT they must 'undo' the transformation to restore equivalence

Goal

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
   (Tabulate (i 6)
     (Rotate [0, 0, 60i]
       (Translate [1,−0.5,0]
         (Cuboid [10, 1, 1]))))))
```

Inverse Transformations

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(Fold Union
 (Map2 Rotate
  (List [0, 0, 120] [0, 0, 0] [0, 0, 300] [0, 0, 180] [0, 0, 240] [0, 0, 60])
  (Repeat 6
   (Translate [1, −0.5, 0]
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Solver permutes the list to find closed form!

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 (Map2 Rotate
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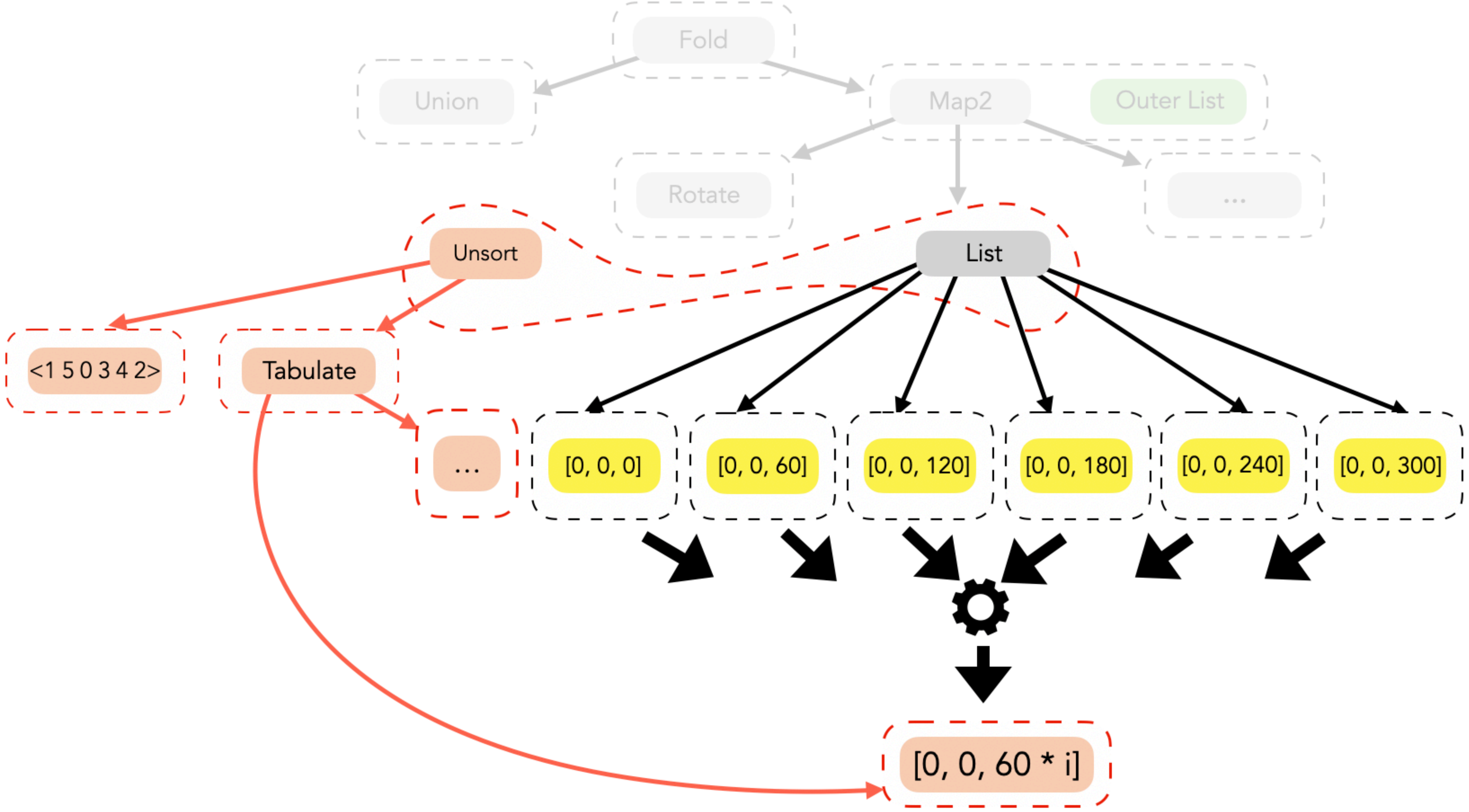
```

Solver annotates the expression with the profitable permutation



If a solver cannot simplify A, but it can simplify f(A) to B, then f^{-1}(B) can be unified with A

Inverse Transformations

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   (Translate [1, −0.5, 0]
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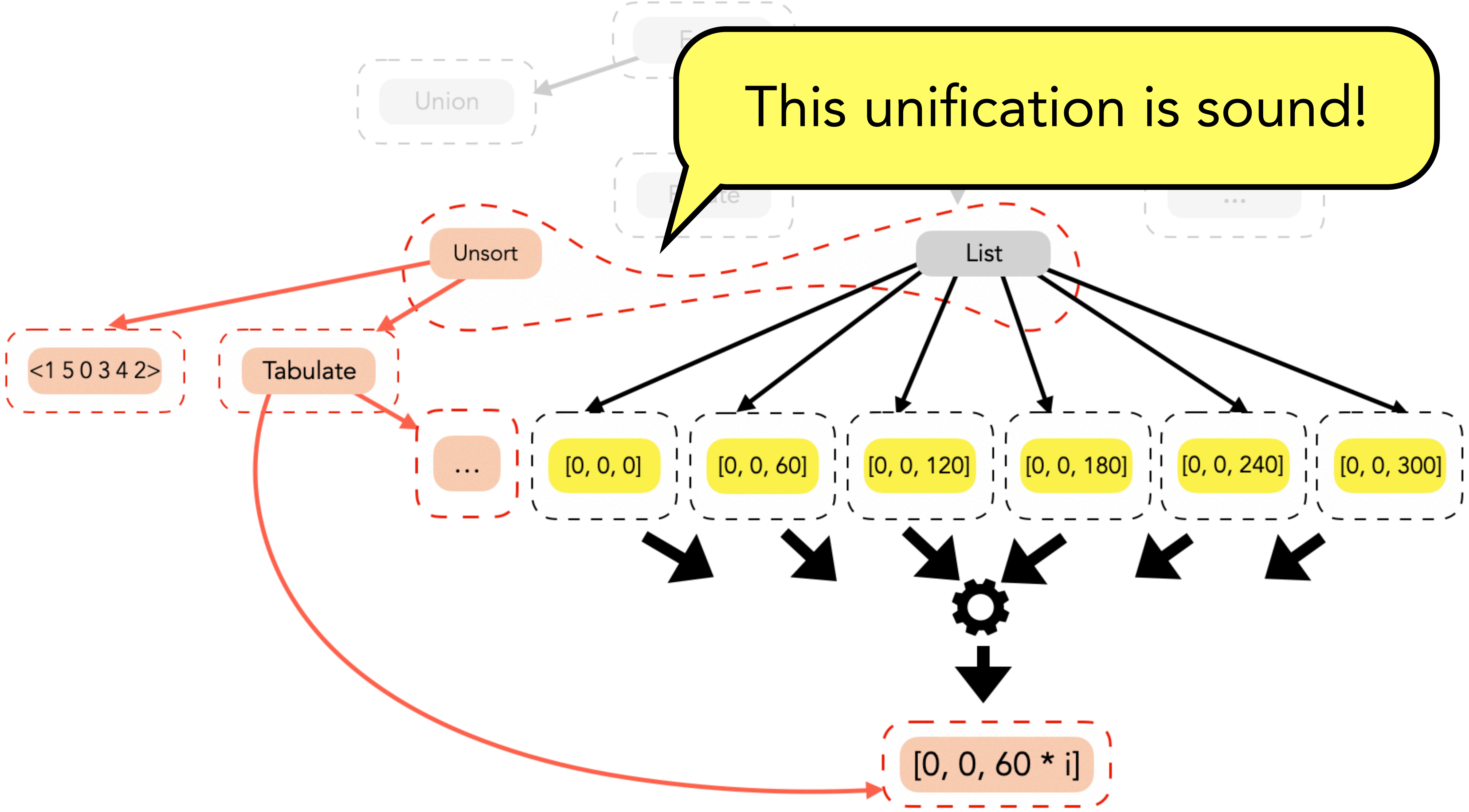
Solver annotates the expression with the profitable permutation



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Inverse Transformations

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    (Cuboid [10, 1, 1])))))
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Goal

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,−0.5,0]
     (Cuboid [10, 1, 1]))))))
```

Inverse Transformations

```
(Fold Union
 (Map2 Rotate
  (Unsort <1 5 0 3 4 2> (Tabulate (i 6) [0, 0, 60 * i]))
  (Repeat 6
   (Translate [1, −0.5, 0]
    (Cuboid [10, 1, 1])))))
```

Propagate and Eliminate

Syntactic rewrites

```
(Fold Union
 (Unsort <1 5 0 3 4 2> (Sort <1 5 0 3 4 2>
 (Map2 Rotate
  (Unsort <1 5 0 3 4 2> (Tabulate (i 6) [0, 0, 60 * i]))
  (Repeat 6
   (Translate [1, −0.5, 0]
    (Cuboid [10, 1, 1])))))))
```

Goal

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,−0.5,0]
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Inverse Transformations

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(Fold Union
 (Map2 Rotate
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Propagate and Eliminate

Syntactic rewrites

```
(Fold Union
 (Unsort <1 5 0 3 4 2> (Sort <1 5 0 3 4 2>
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  (Repeat 6
   (Translate [1, −0.5, 0]
    (Cuboid [10, 1, 1])))))))
```

Effectively a no-op, but allows sorting the concrete list equivalent to **Map2**

Goal

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,−0.5,0]
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Inverse Transformations

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(Fold Union
 (Map2 Rotate
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  (Repeat 6
   (Translate [1, −0.5, 0]
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```

Propagate and Eliminate

Syntactic rewrites

```
(Fold Union
 (Unsort <1 5 0 3 4 2> (Sort <1 5 0 3 4 2>
 (Map2 Rotate
  (Unsort <1 5 0 3 4 2> bulate (i 6) [0, 0, 60 * i]))
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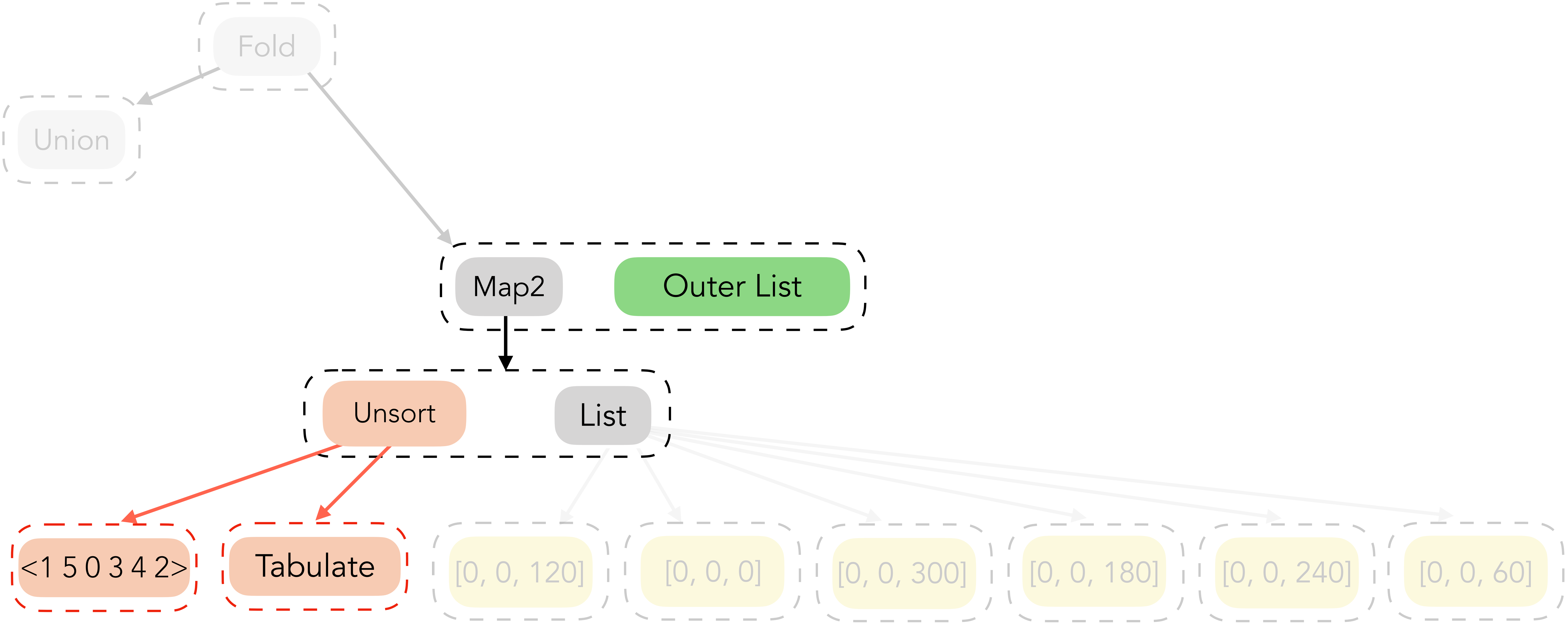

Goal

Inverse Transformations

```
(Fold Union
 (Map2 Rotate
  (Unsort <1 5 0 3 4 2> (Tabulate (i 6) [0, 0, 60 * i]))
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```

Propagate and Eliminate

→

Syntactic rewrites

```
(Fold Union
 (Unsort <1 5 0 3 4 2> (Sort <1 5 0 3 4 2>
 (Map2 Rotate
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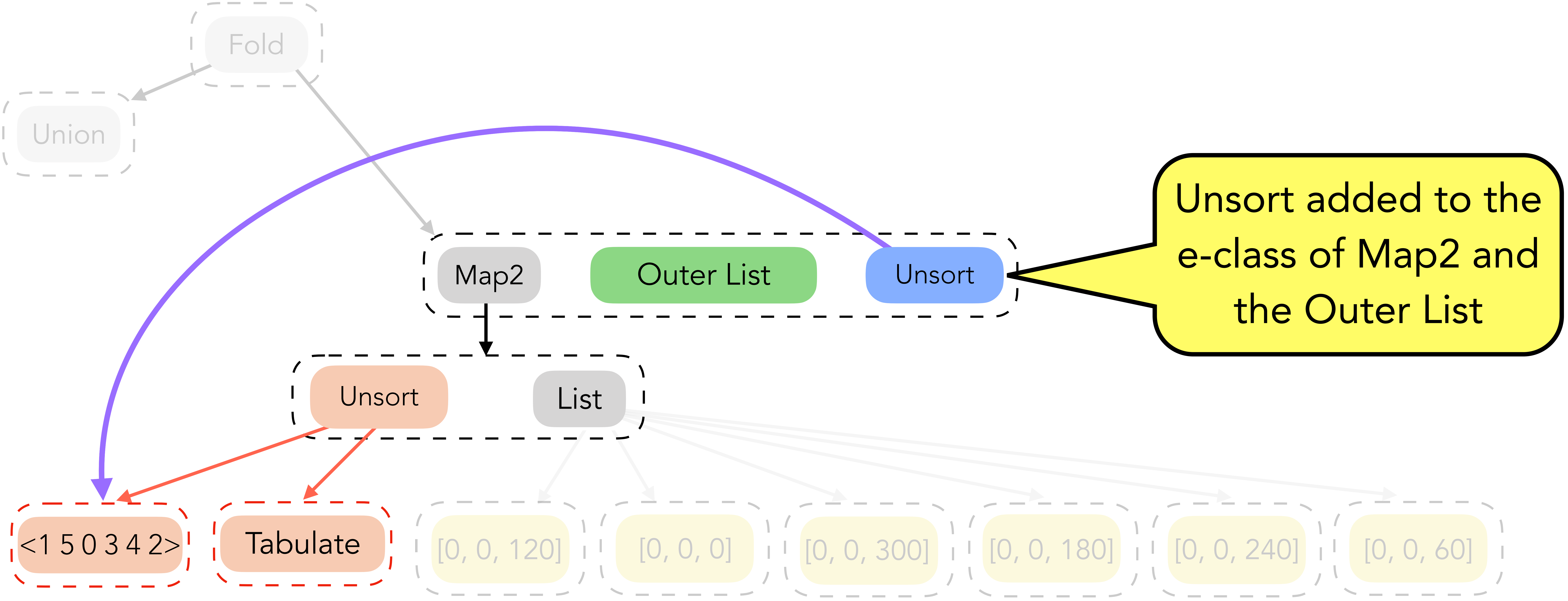

Goal

```
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 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,−0.5,0]
     (Cuboid [10, 1, 1]))))))
```

Inverse Transformations

```
(Fold Union
 (Map2 Rotate
  (Unsort <1 5 0 3 4 2> (Tabulate (i 6) [0, 0, 60 * i]))
  (Repeat 6
   (Translate [1, –0.5, 0]
    (Cuboid [10, 1, 1])))))
```

Propagate and Eliminate

Syntactic rewrites

```
(Fold Union
 (Unsort <1 5 0 3 4 2> (Sort <1 5 0 3 4 2>
 (Map2 Rotate
  (Unsort <1 5 0 3 4 2> bulate (i 6) [0, 0, 60 * i]))
  (Repeat 6
   (Translate [1, –0.5, 0]
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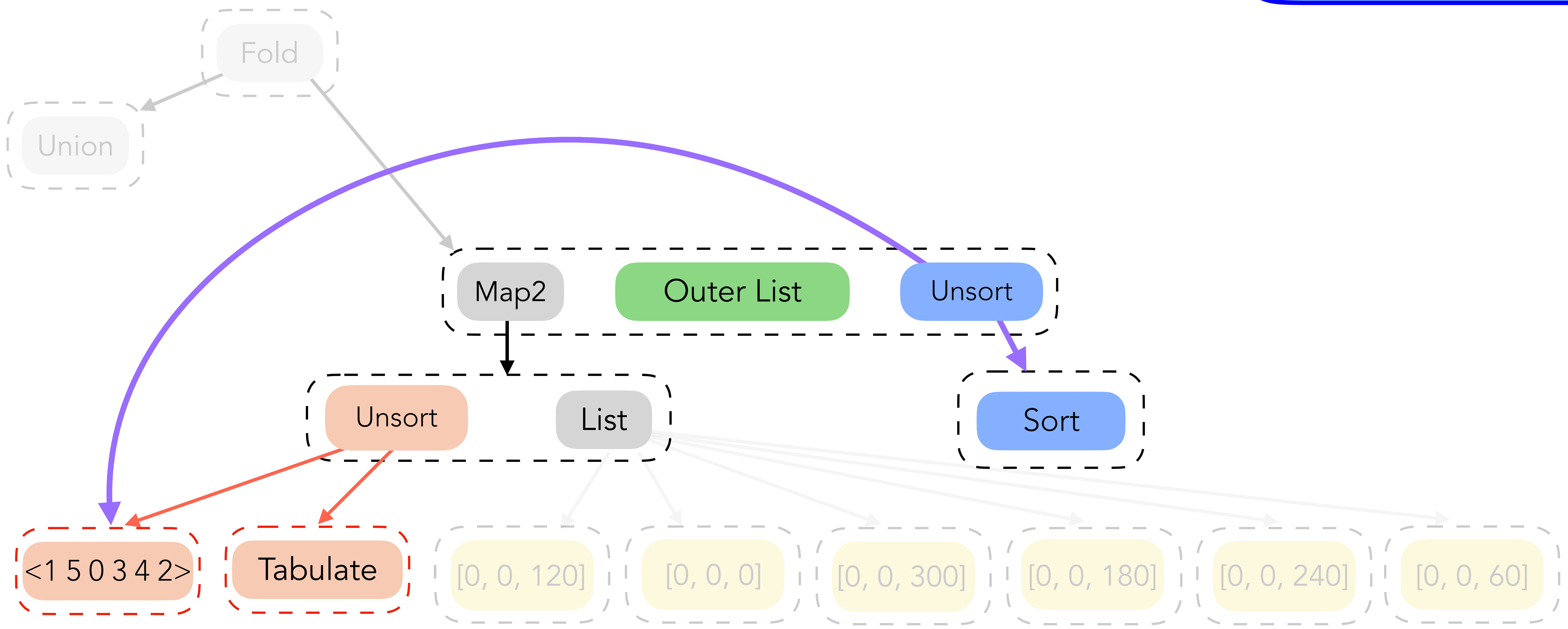

Goal

```
(Union
 (Cylinder [1, 5, 5])
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  (Tabulate (i 6)
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Inverse Transformations

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(Fold Union
 (Map2 Rotate
  (Unsort <1 5 0 3 4 2> (Tabulate (i 6) [0, 0, 60 * i]))
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   (Translate [1, −0.5, 0]
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Propagate and Eliminate

Syntactic rewrites

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(Fold Union
 (Unsort <1 5 0 3 4 2> (Sort <1 5 0 3 4 2>
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  (Unsort <1 5 0 3 4 2> bulate (i 6) [0, 0, 60 * i]))
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   (Translate [1, −0.5, 0]
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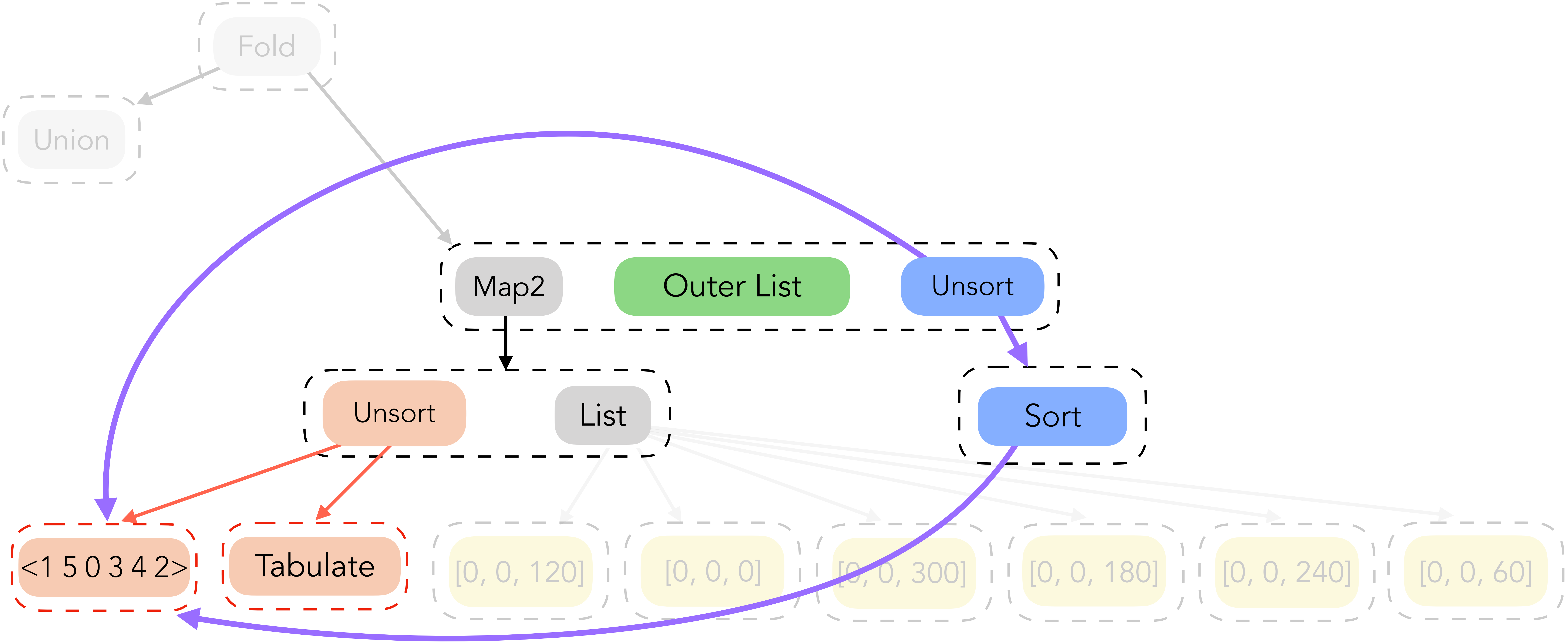

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```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
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(Fold Union
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Propagate and Eliminate

Syntactic rewrites

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(Fold Union
 (Unsort <1 5 0 3 4 2> (Sort <1 5 0 3 4 2>
 (Map2 Rotate
  (Unsort <1 5 0 3 4 2> bulate (i 6) [0, 0, 60 * i]))
  (Repeat 6
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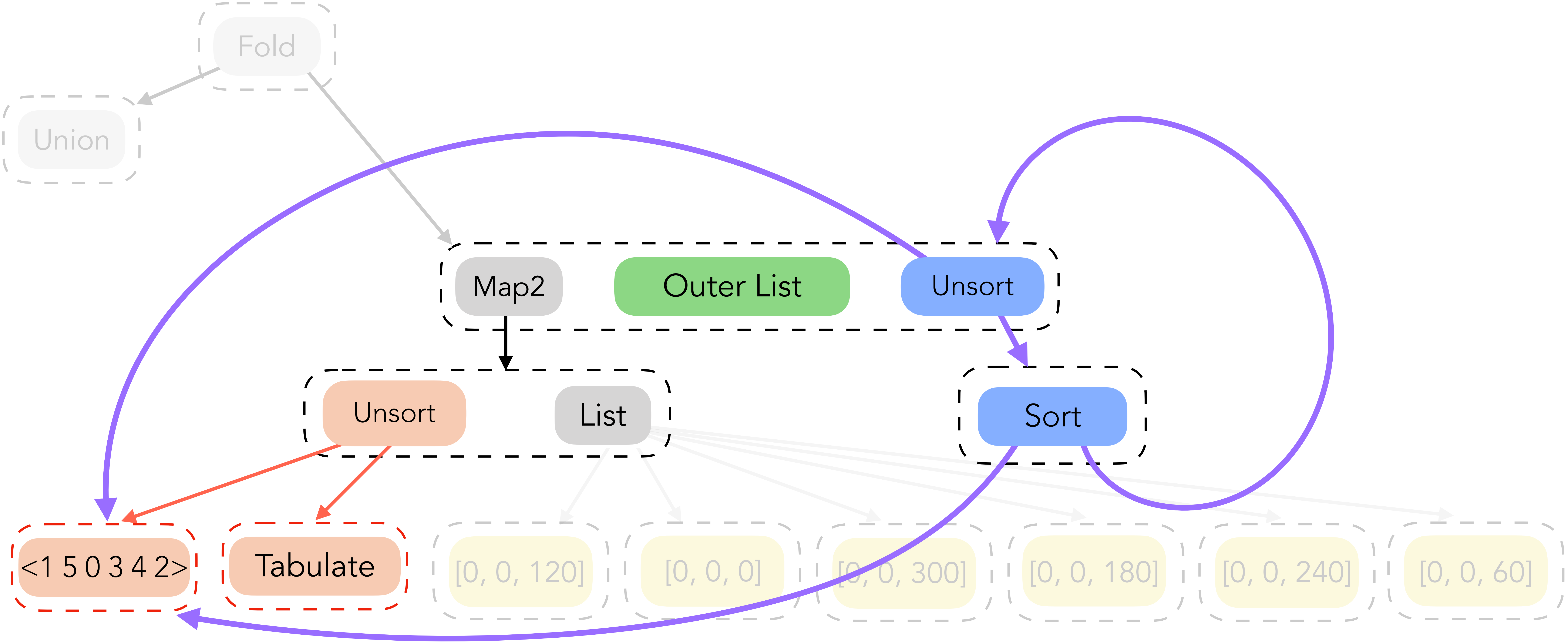
Effectively a no-op, but allows sorting the concrete list equivalent to **Map2**



Goal

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
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Inverse Transformations

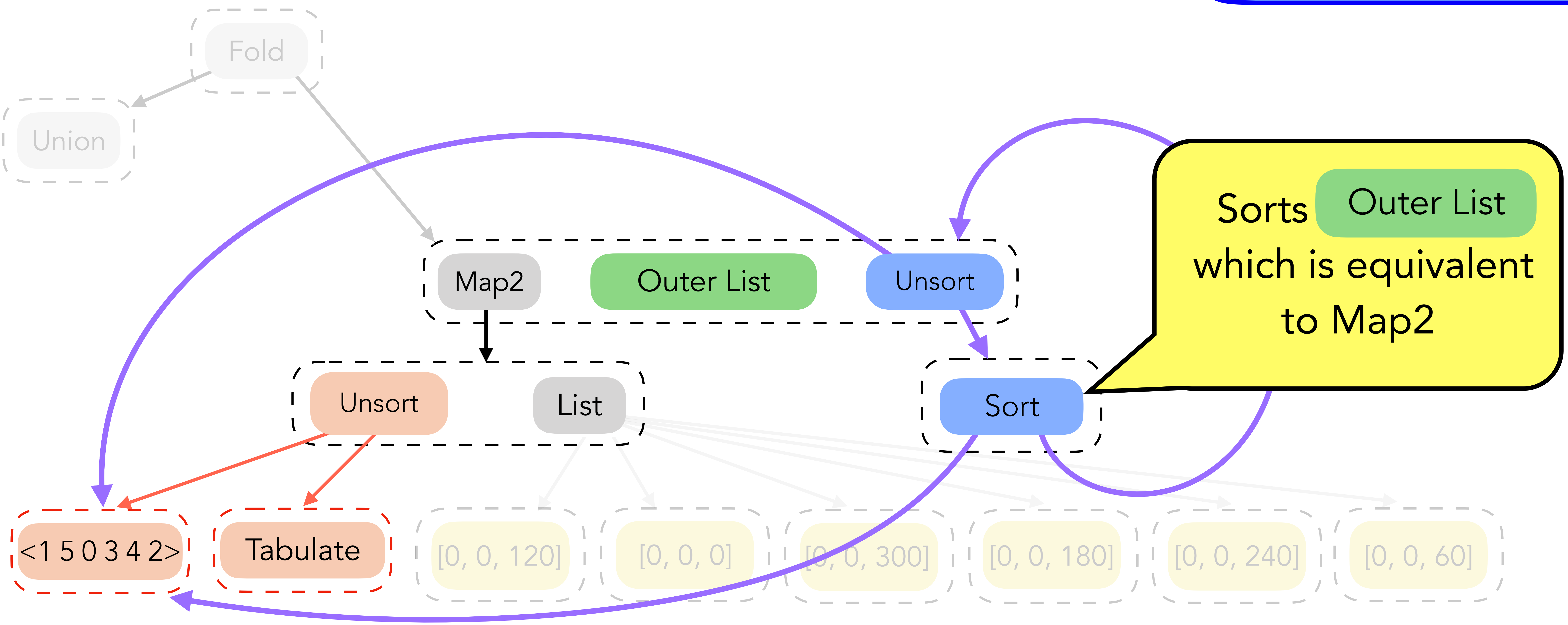
(Fold Union
 (Map2 Rotate
  (**Unsort <1 5 0 3 4 2>** (Tabulate (i 6) [0, 0, 60 * i]))
  (Repeat 6
   (Translate [1, –0.5, 0]
    (Cuboid [10, 1, 1])))))

Propagate and Eliminate

Syntactic rewrites

(Fold Union
 (**Unsort <1 5 0 3 4 2> (Sort <1 5 0 3 4 2>**
 (Map2 Rotate
  (**Unsort <1 5 0 3 4 2>** (Tabulate (i 6) [0, 0, 60 * i]))
  (Repeat 6
   (Translate [1, –0.5, 0]
    (Cuboid [10, 1, 1])))))))

Effectively a no-op, but allows sorting the concrete list equivalent to **Map2**



Goal

(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
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Inverse Transformations

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(Fold Union
 (Map2 Rotate
  (Unsort <1 5 0 3 4 2> (Tabulate (i 6) [0, 0, 60 * i]))
  (Repeat 6
   (Translate [1, –0.5, 0]
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```

Propagate and Eliminate

→

Syntactic rewrites

```
(Fold Union
 (Unsort <1 5 0 3 4 2> (Sort <1 5 0 3 4 2>
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  (Unsort <1 5 0 3 4 2> (Ta bulate (i 6) [0, 0, 60 * i]))
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   (Translate [1, –0.5, 0]
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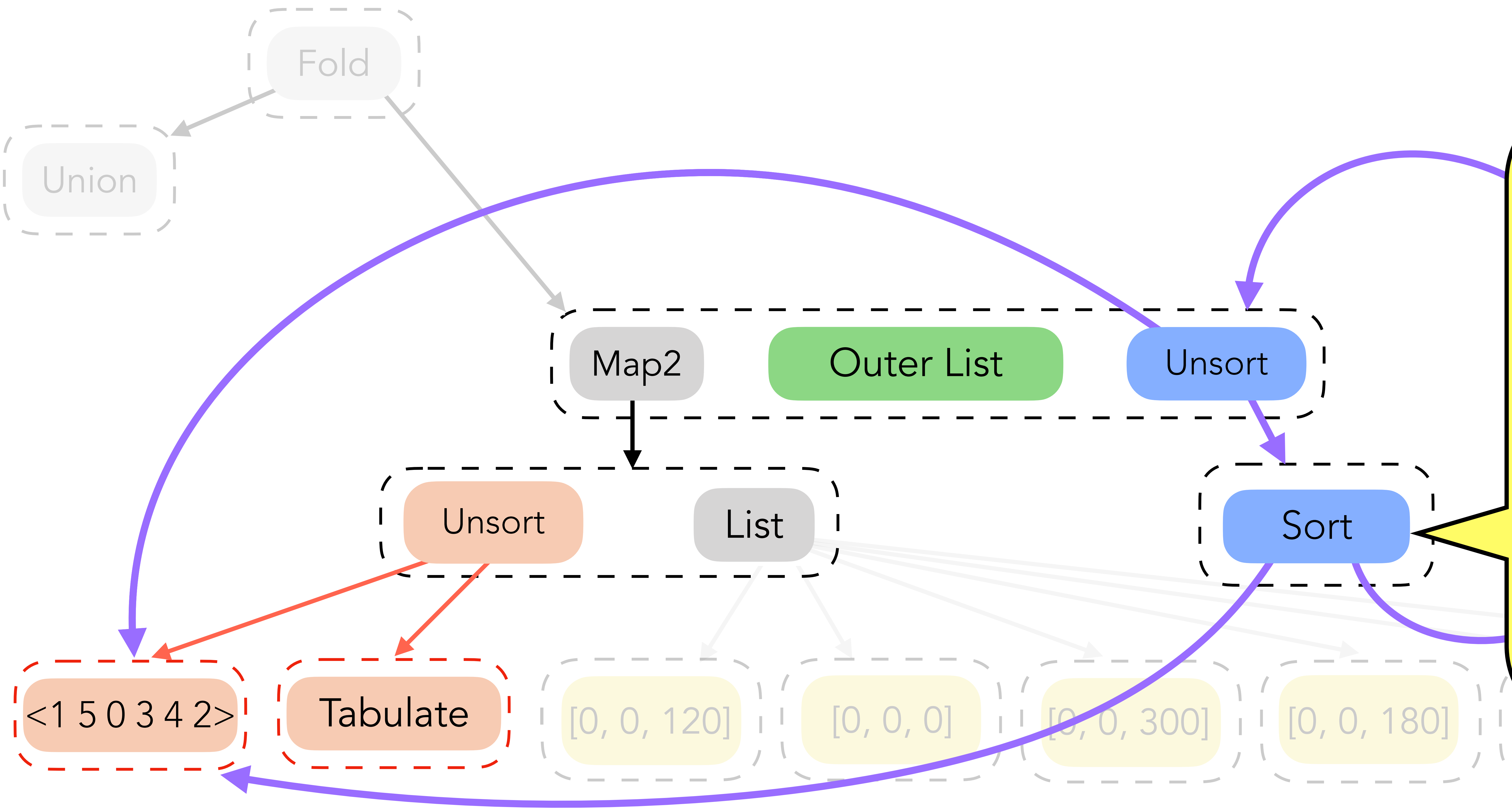
Effectively a no-op, but allows sorting the concrete list equivalent to **Map2**



Structure Finder and Custom Solvers apply on this sorted list

Goal

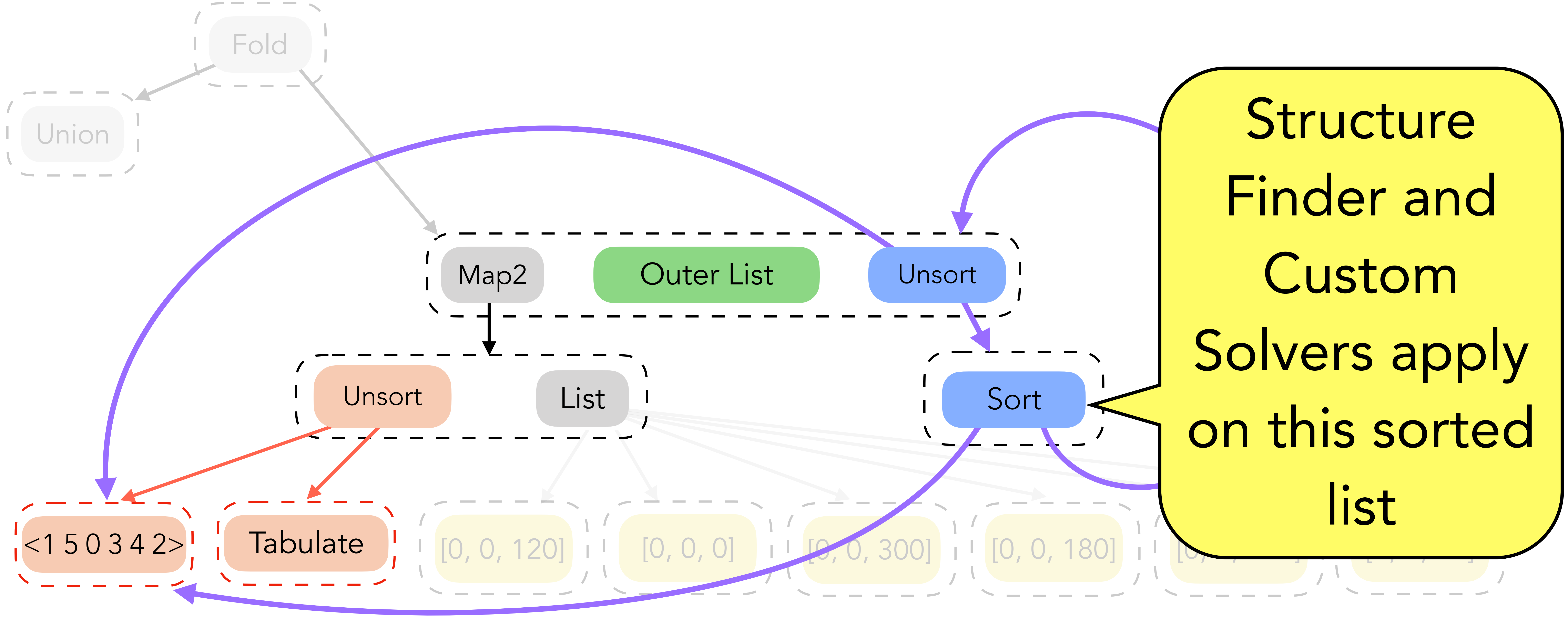
```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,–0.5,0]
     (Cuboid [10, 1, 1]))))))
```

Inverse Transformations

```
(Fold Union
  (Unsort <1 5 0 3 4 2> (Sort <1 5 0 3 4 2>
  (Map2 Rotate
    (Unsort <1 5 0 3 4 2> (Tabulate (i 6) [0, 0, 60 * i]))
    (Repeat 6
      (Translate [1, −0.5, 0]
        (Cuboid [10, 1, 1])))))))
```

Custom solvers on the sorted outer list

```
(Fold Union
  (Unsort <1 5 0 3 4 2>
    (Tabulate (i 6)
      (Rotate [0, 0, 60 * i])
        (Translate [1, −0.5, 0]
          (Cuboid [10, 1, 1]))))))
```



Goal

```
(Union
  (Cylinder [1, 5, 5])
  (Fold Union
    (Tabulate (i 6)
      (Rotate [0, 0, 60i]
        (Translate [1,–0.5,0]
          (Cuboid [10, 1, 1]))))))
```

Inverse Transformations

```
(Fold Union
 (Unsort <1 5 0 3 4 2> (Sort <1 5 0 3 4 2>
 (Map2 Rotate
  (Unsort <1 5 0 3 4 2> (Tabulate (i 6) [0, 0, 60 * i]))
  (Repeat 6
   (Translate [1, −0.5, 0]
    (Cuboid [10, 1, 1])))))))
```

Custom solvers on the sorted outer list

```
(Fold Union
 (Unsort <1 5 0 3 4 2>
  (Tabulate (i 6)
   (Rotate [0, 0, 60 * i])
    (Translate [1, −0.5, 0]
     (Cuboid [10, 1, 1]))))))
```

Fold Union is invariant to list order

Syntactic rewrite to eliminate Unsort

Goal

```
(Union
 (Cylinder [1, 5, 5])
 (Fold Union
  (Tabulate (i 6)
   (Rotate [0, 0, 60i]
    (Translate [1,−0.5,0]
     (Cuboid [10, 1, 1]))))))
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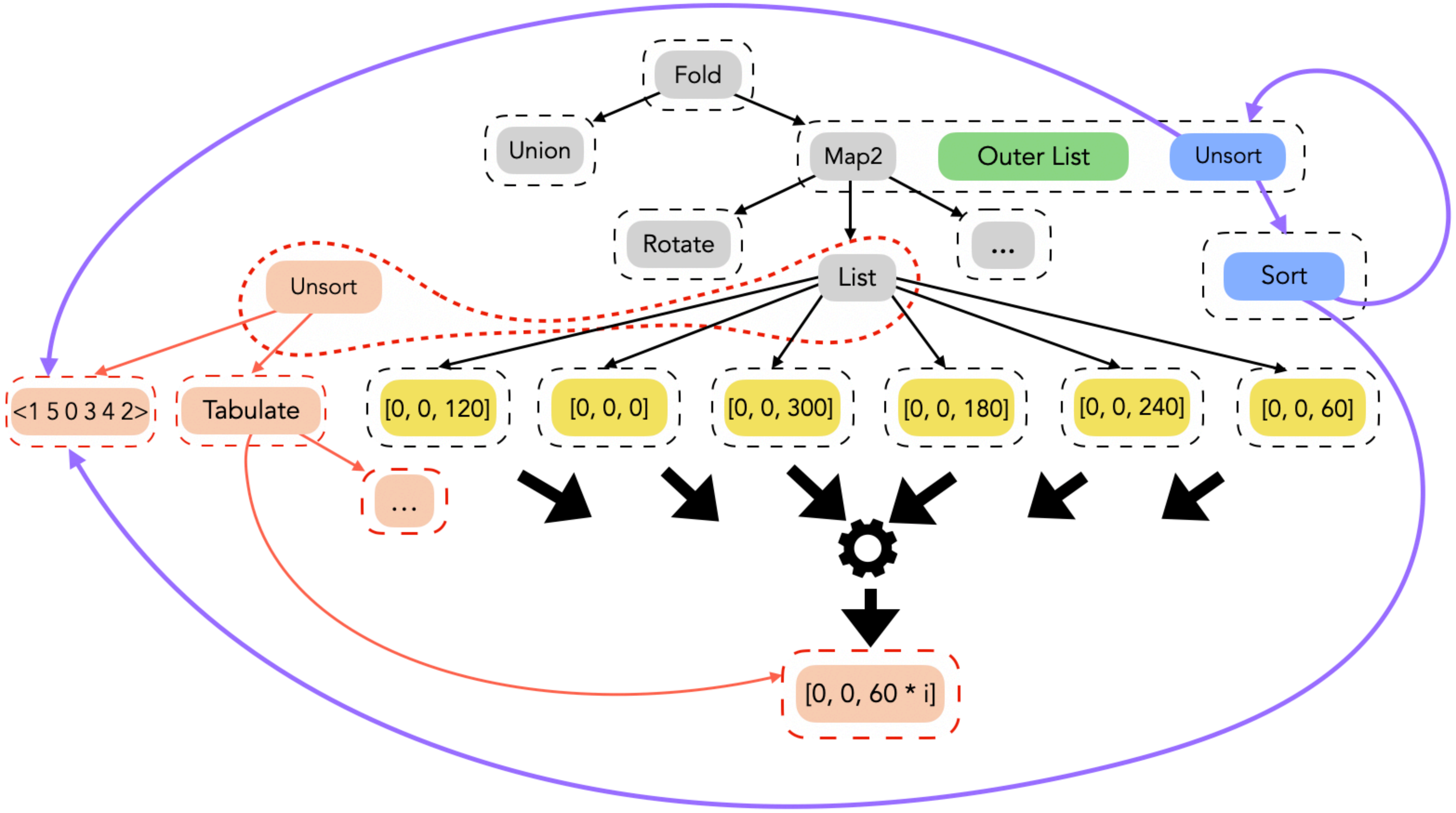


Inverse Transformations

Example transformations: sorting, partitioning, cartesian-to-spherical

Rewrites applied until saturation (or timeout) and a cost function (AST size) used to extract best program

Implementation

~ 2000 LOC in Rust

65 rewrites

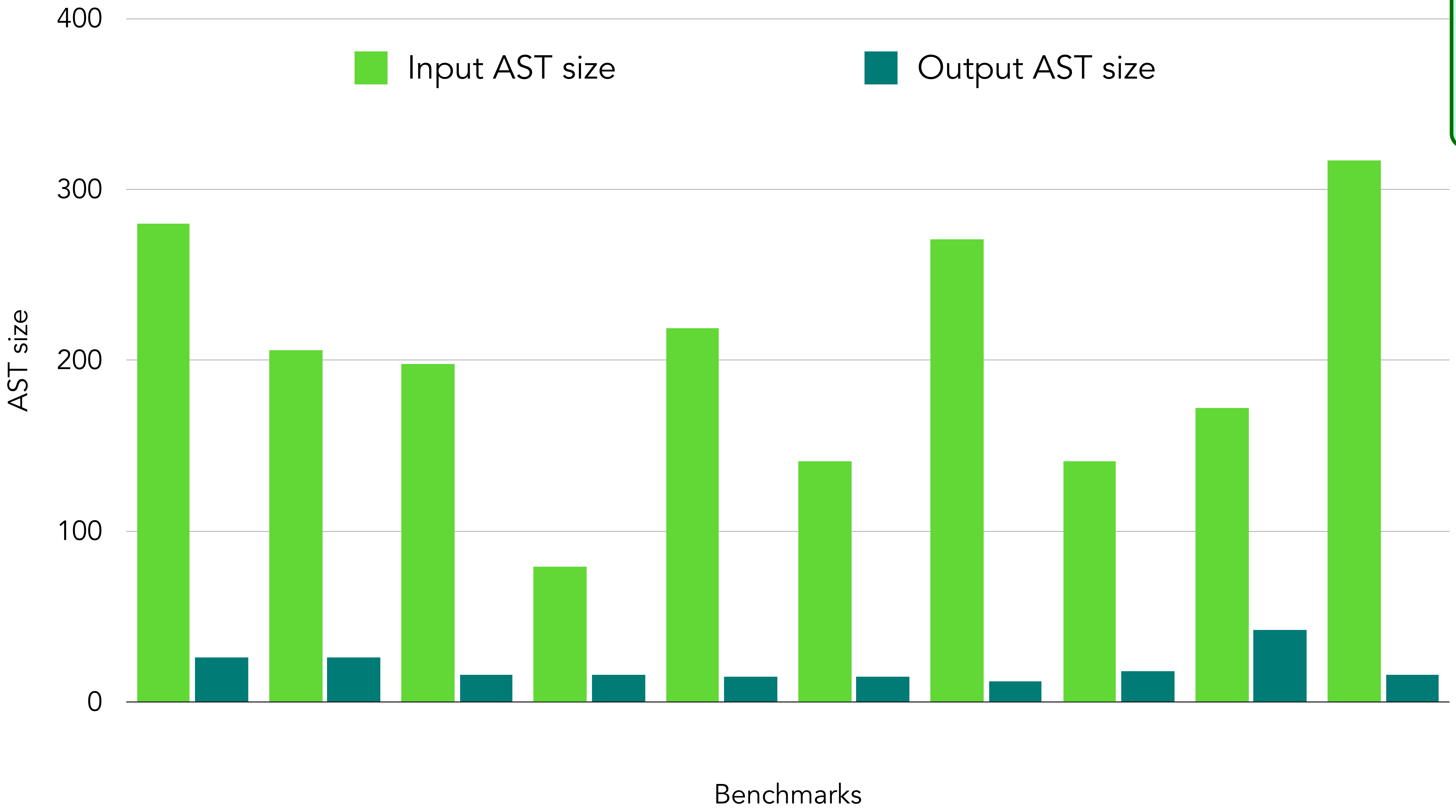
https://github.com/uwplse/szalinski

Uses the Egg E-graph library: https://github.com/mwillsey/egg

Talk to Max about Egg!

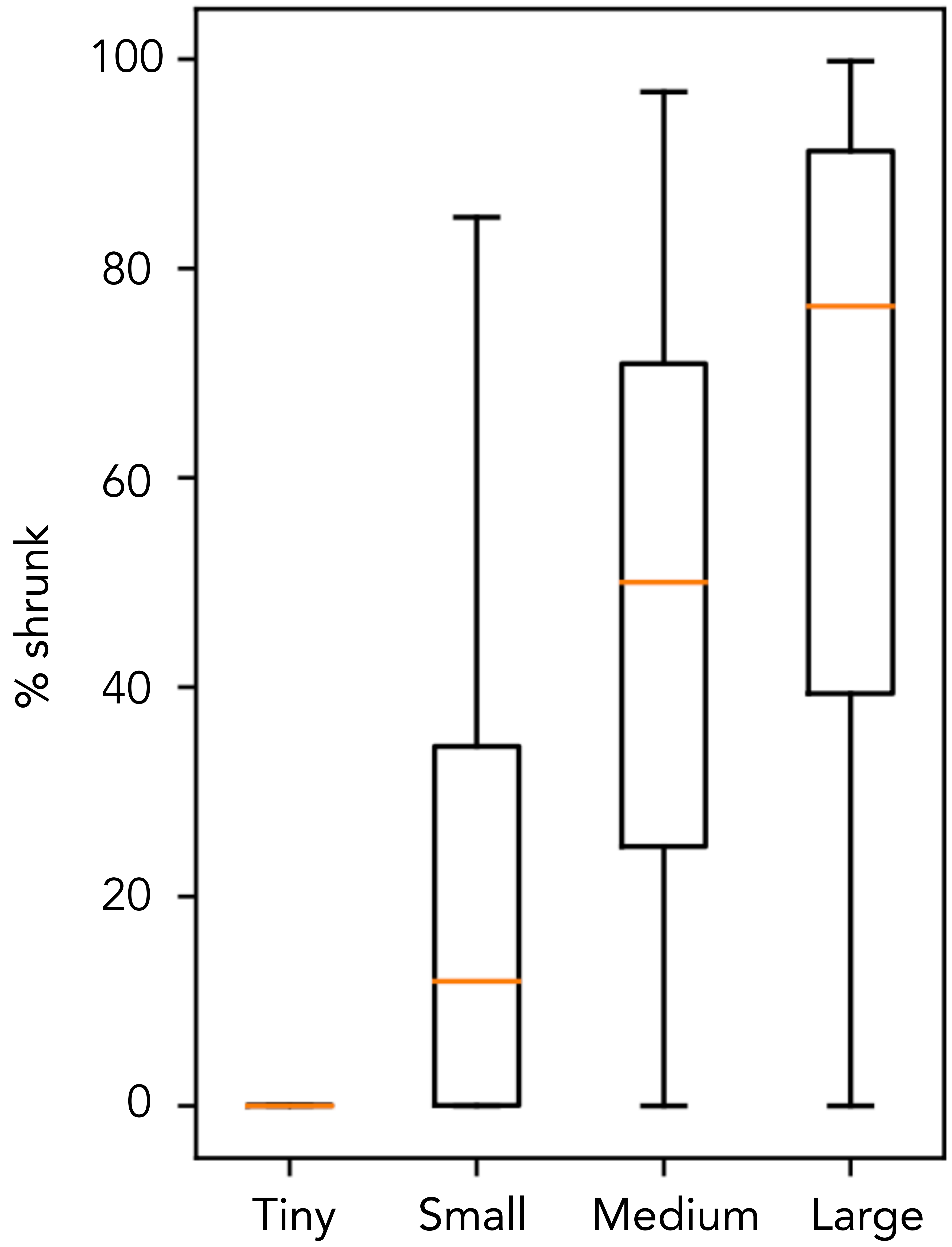
End-to-End Evaluation

Results of running Szalinski on outputs of Reincarnate*



* [ICFP 2018]

Scalability



2127 programs from Thingiverse

Tiny: AST size < 30

Small: 30 < AST size < 100

Medium: 100 < AST size < 300

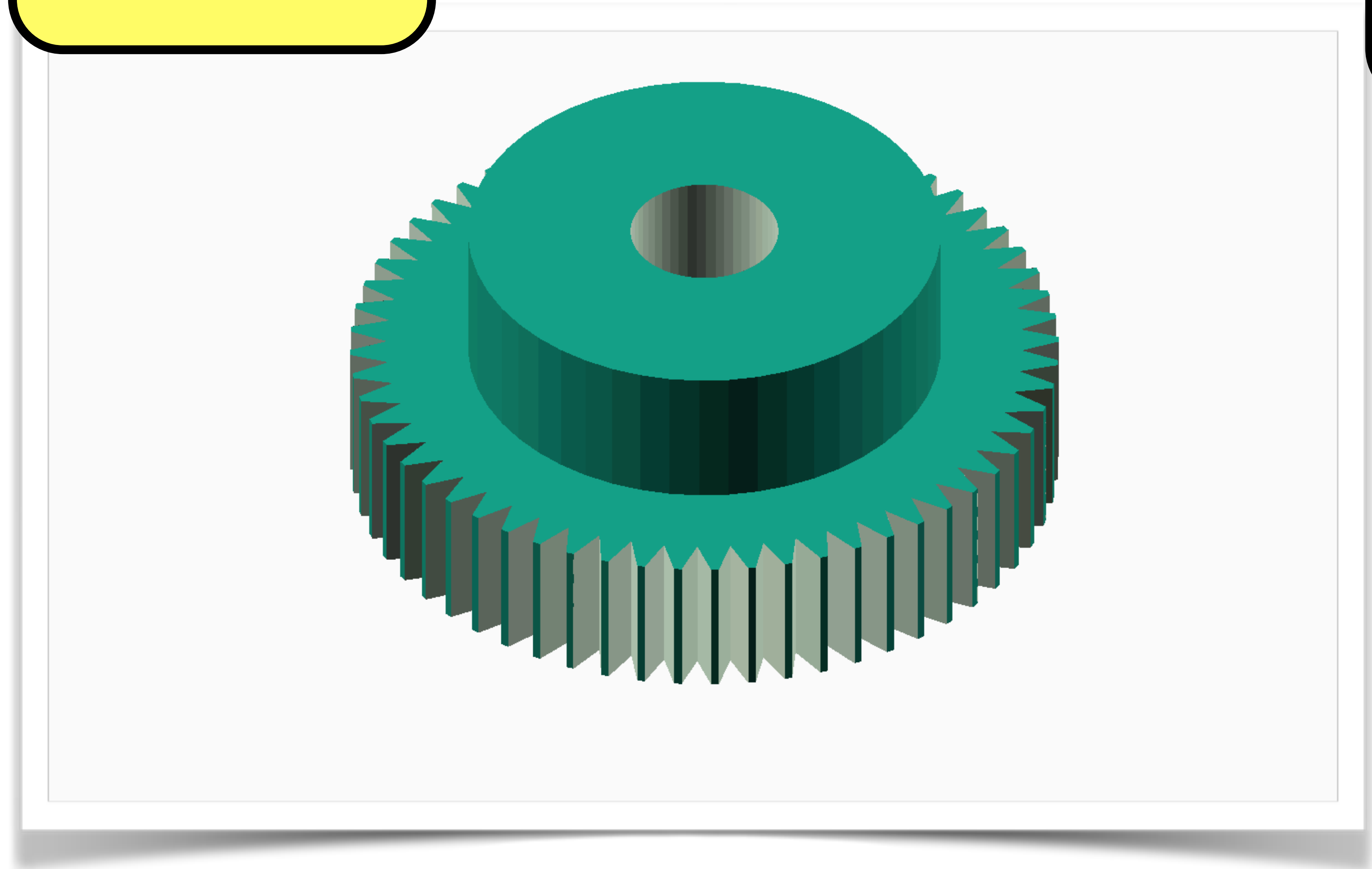
Large: AST size > 300

Larger programs shrink more

< 1 second

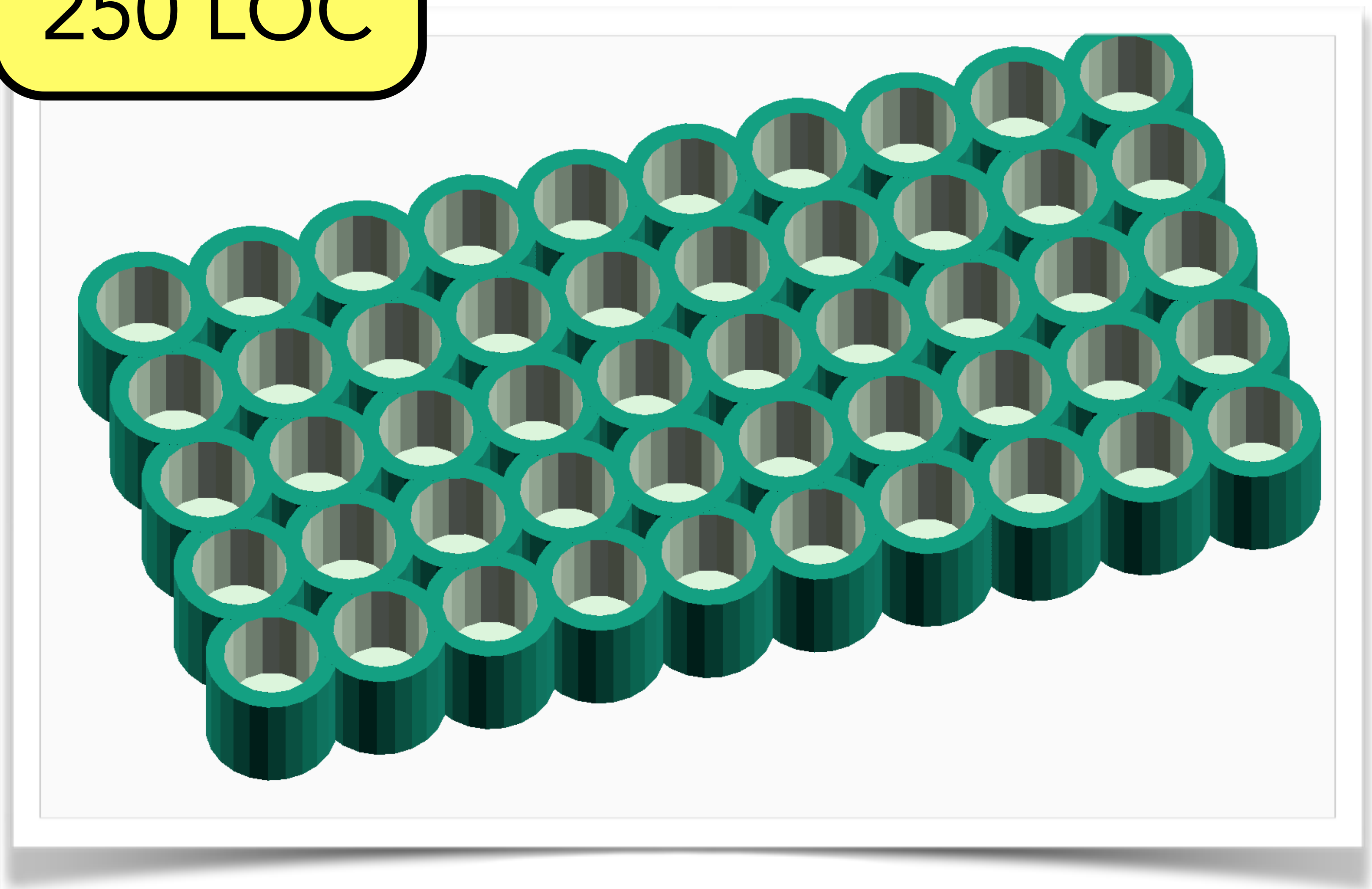
Examples

350 LOC



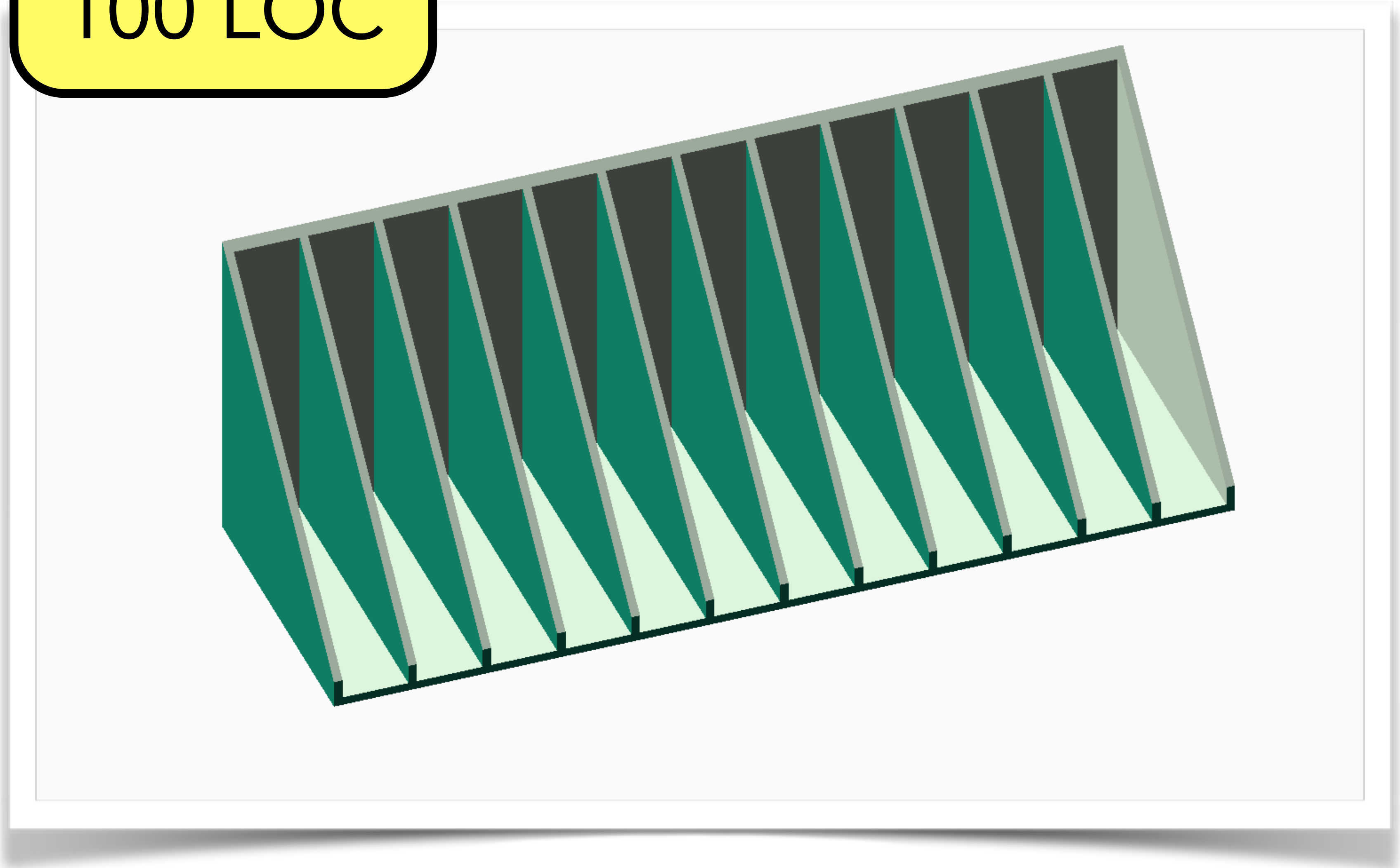
```
(Fold Difference
 (List (Union
    (Cylinder [100, 80, 80])
    (Cylinder [50, 120, 120]))
  (Translate [0, 0, -1] (Cylinder [102, 25, 25]))
  (Fold Union (Tabulate (i 60)
    (Rotate [0, 0, 6 * i]
     (Translate [125, 0, 0]
      (Scale [2.5, 1, 1]
       (Rotate [0, 0, 45]
        (Translate [0, 0, 25]
          (Cuboid [10, 10, 52]))))))))))
```

250 LOC



```
(Fold Union
 (Tabulate (i 10) (j 5)
  (Translate
    [12.2 * i + 12.2, 12.2 * j + 12.2, 0]
   (Difference
    (Cylinder [13, 7.1, 7.1])
    (Translate [0, 0, 3]
      (Cylinder [11, 5.1, 5.1]))))))
```

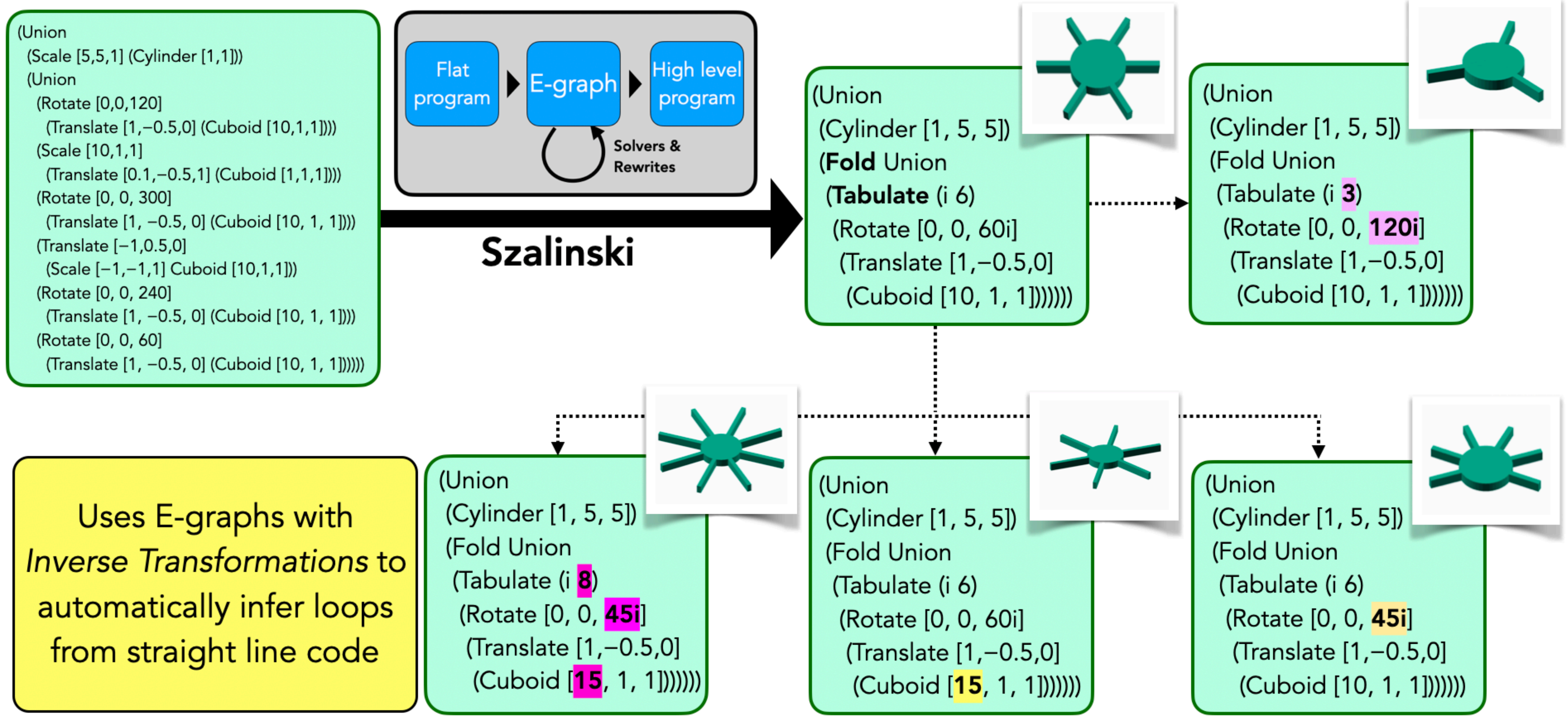
100 LOC



```
(Fold Union
 (Tabulate (i 12)
  (Translate [0, 13* i, 0]
   (Fold Difference
    (List
     (Cuboid [53.1 14.5 58])
     (Translate [1.5, 1.5, 1.5]
       (Cuboid [51.6, 11.5, 56.6]))
     (Translate [0 0 58]
      (Rotate [0, 45, 0]
       (Cuboid [101.5, 14.5, 100]))))))))
```

Szalinski: A Tool for Synthesizing Structured CAD Models with Equality Saturation and Inverse Transformations

https://github.com/uwplse/szalinski



Inverse Transformations with E-graphs to find concise, structured programs in < 1 second

Chandrakana Nandi, Max Willsey, Adam Anderson, James R. Wilcox, Eva Darulova, Dan Grossman, Zachary Tatlock